Phase-slip avalanches in the superflow of ^{4}He through arrays of nano-apertures

David Pekker, Roman Barankov & Paul M. Goldbart
Department of Physics & Seitz Materials Research Laboratory
University of Illinois at Urbana-Champaign

- Pekker, Barankov & PMG, *cond-mat/0606560*

PASI, Mar Del Plata, December 2006

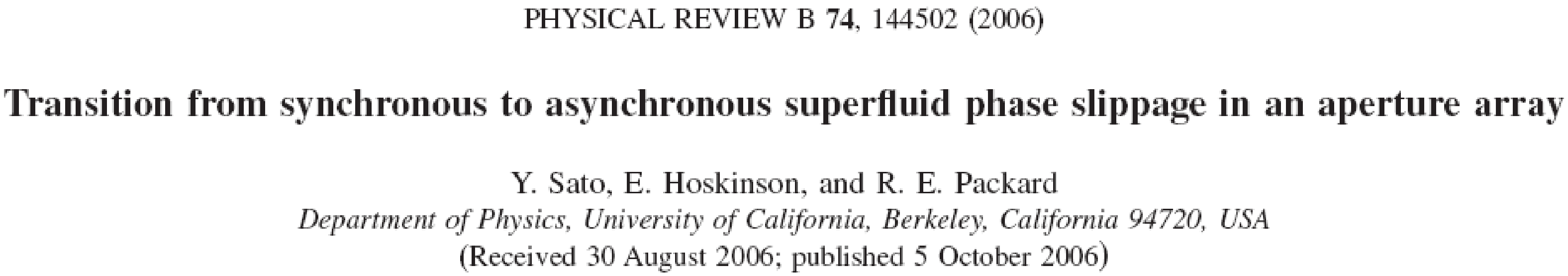
Broad view

- Themes…
  - macroscopic quantum phenomena
  - superfluid Josephson physics & S(f)QUIDs

    dissipative & reversible regimes
  - interactions & disorder

    & their competition
  - non-equilibrium phase transitions
  - *(applications: precision rotation sensors?)*

- Motivation: Packard group's experiments...



PHYSICAL REVIEW B **74**, 144502 (2006)

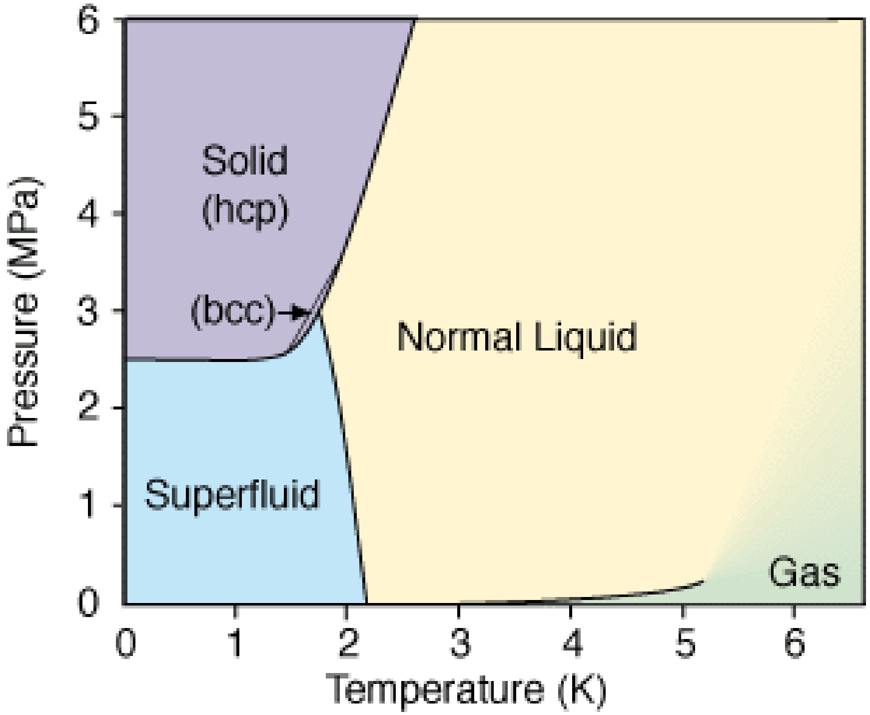
Transition from synchronous to asynchronous superfluid phase slippage in an aperture array

Y. Sato, E. Hoskinson, and R. E. Packard

Department of Physics, University of California, Berkeley, California 94720, USA

(Received 30 August 2006; published 5 October 2006)

Superfluidity of Helium-four



http://ltl.tkk.fi/research/theory/helium.html

New eq’m state below ~ 2K

- Ordered liquid: macro occup of one single-particle quantum state
- New thermodynamic coord: *phase* *(analog of elastic displacement; but longitudinal only)*

$$\sqrt{n_\mathrm{s}}\,\mathrm{e}^{i\phi(\mathbf{r})} = \langle\widehat{\psi}(\mathbf{r})\rangle_{\mathrm{eq}}$$

- Phase gradients = eq’m currents (supercurrents)

$$\mathbf{v} = (\hbar/m)\nabla\phi$$

- Quantal origin; quantal dynamics

Sensing rotation

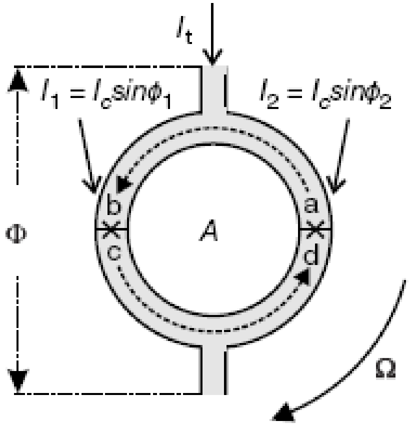
- Superfluid ^{3}He S(f)QUID
  - magnetic flux → rotation flux

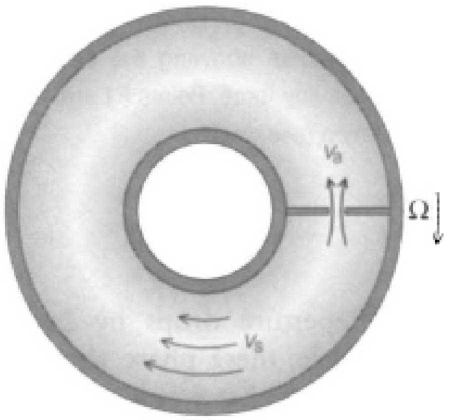
$$\frac{\mathbf{B}\cdot\mathbf{A}}{\Phi_0(=hc/2e)} \longrightarrow \frac{\mathbf{\Omega}\cdot\mathbf{A}}{\kappa(=h/2m_3)}$$

  - critical current: $I_{\mathrm{c,sys}} = 2I_{\mathrm{c}}\left|\cos(2\pi\mathbf{\Omega}\cdot\mathbf{A}/\kappa)\right|$
  - exercise: rotating frame?
    Lagrangian picks up Lorentz-like $m\,\dot{\mathbf{r}}\cdot(\mathbf{\Omega}\times\mathbf{r})$

- Superfluid ^{4}He rotation sensor
  - measure flow rate through aperture
  - infer Earth's rotation rate

Simmonds et al., Nature 412, 55 (2001)



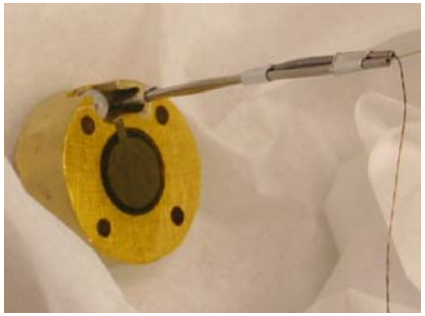


Schwab et al., Nature 386, 585 (1997)

Berkeley group experiment (Sato et al. ’06)

fridge

helium cell



flexible diaphragm

aperture array (center)

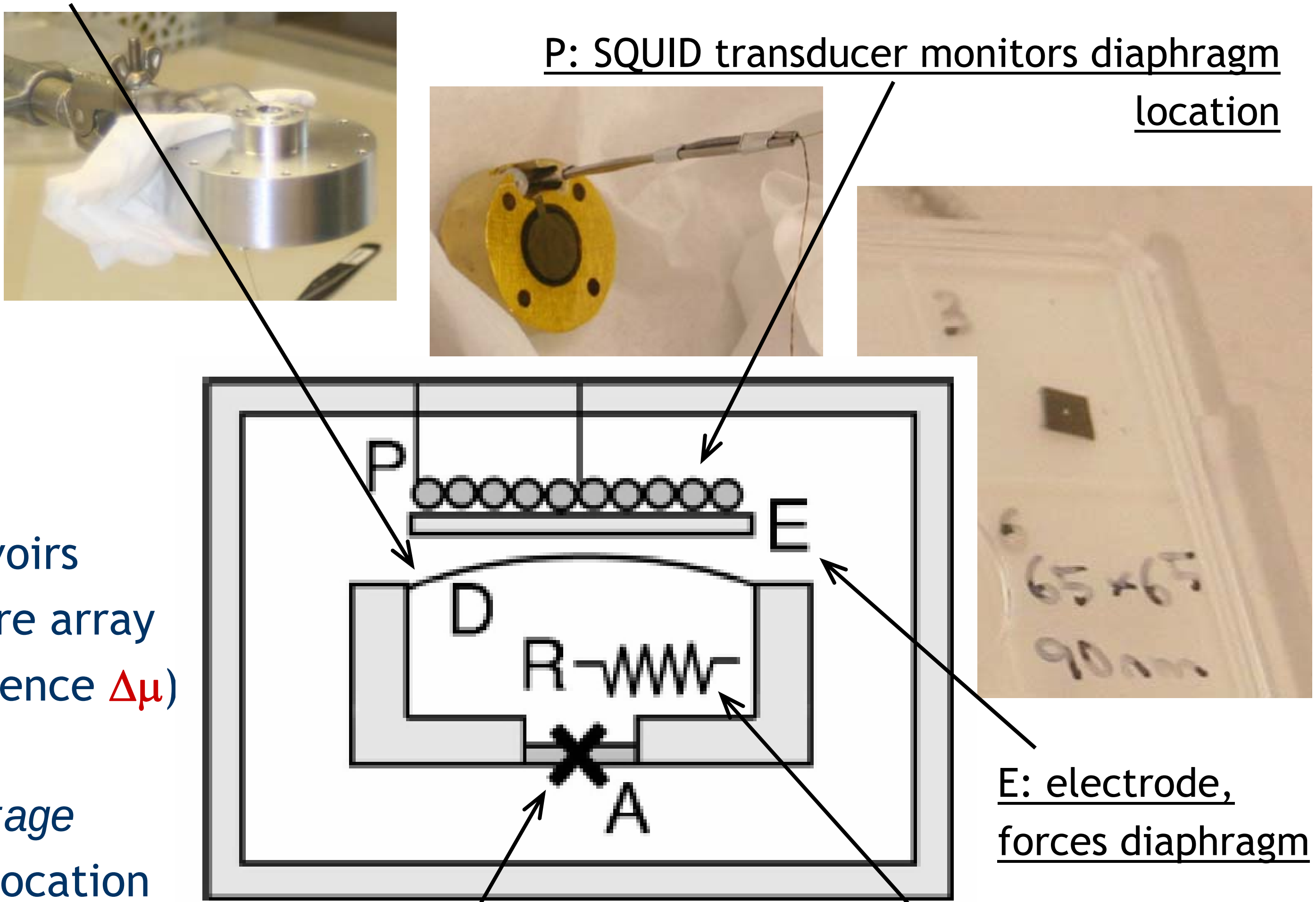
Berkeley group experiment (Sato et al. ’06)

- two superfluid reservoirs
- connected by aperture array
- control ΔP and ΔT (hence $\Delta\mu$) between reservoirs

  analog of voltage
- measure diagram location
- infer current through nano-apertures I

D: diaphragm changes reservoir volumes

P: SQUID transducer monitors diaphragm location



E: electrode, forces diaphragm

A: aperture array

R: heater

Berkeley group experiment caricature

Two superfluid reservoirs…

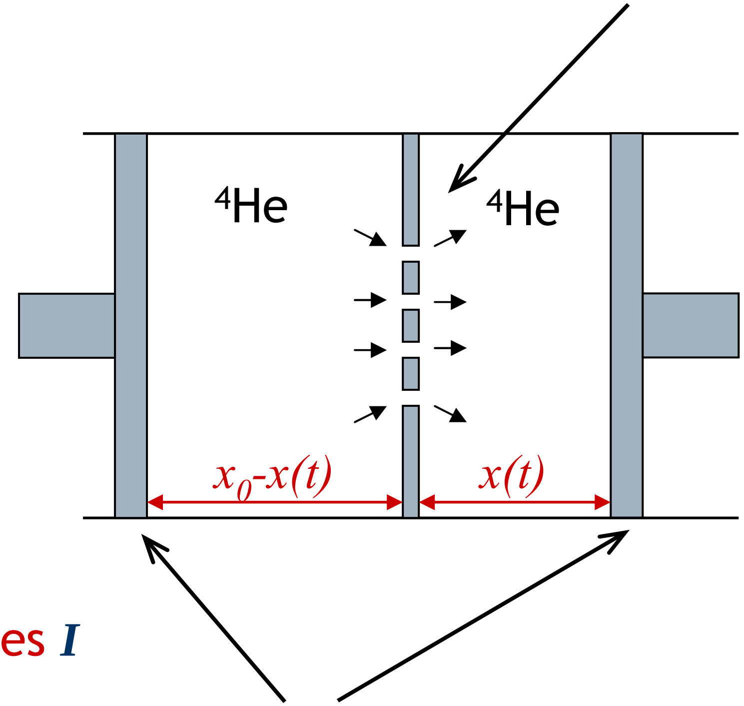
connected by nano-aperture array

- control Δμ between reservoirs

  (analog of voltage)
- measure piston locations

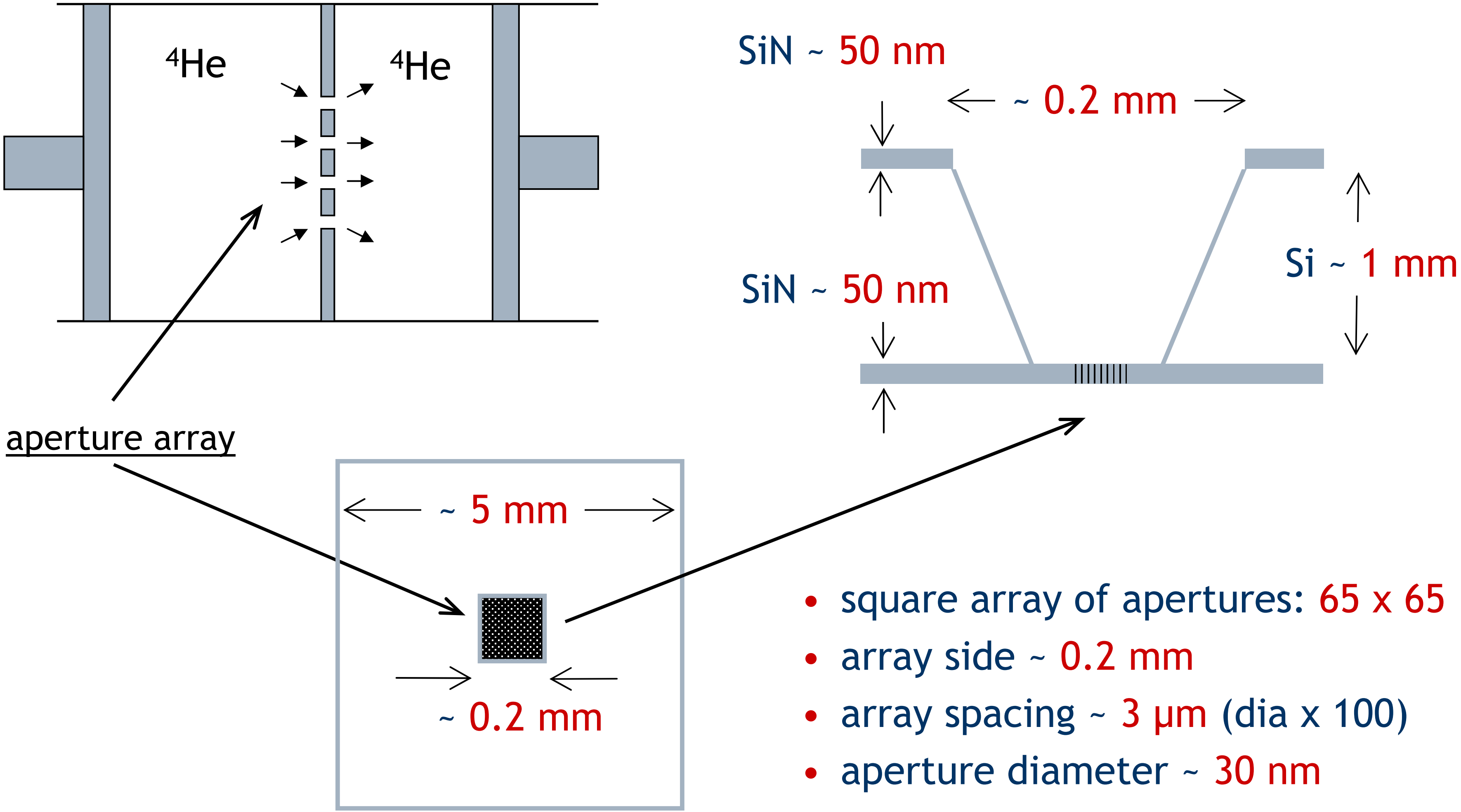
  (constrained)
- infer current through nano-apertures ***I***

aperture array



Pistons: pressure controls Δμ
Location: measures reservoir volume
& hence current

Nano-aperture array geometry



- square array of apertures: 65 x 65
- array side ~ 0.2 mm
- array spacing ~ 3 μm (dia x 100)
- aperture diameter ~ 30 nm
- aperture length ~ 50 nm

Outline

- What's found in the Berkeley group experiments?
- Back-of-the-envelope scenario
- Elements of a model
- How does the model behave
- Other avalanching systems
- Implications for future experiments (& rotation sensor design?)
- Concluding remarks

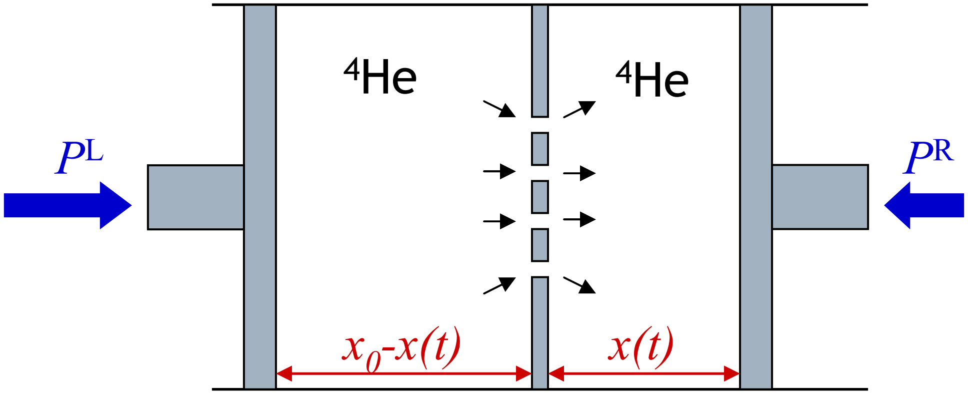
Expectations for the Berkeley group experiments I

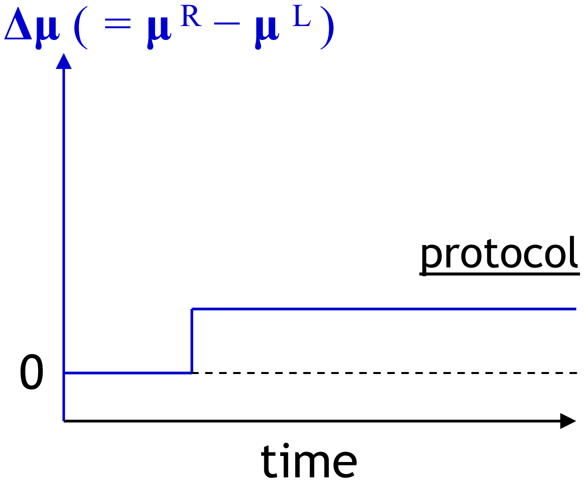
- Seek AC Josephson-type phenomena
  - apply constant Δμ
  - measure total current power

    $$|I(\omega_{\mathrm{J}})|^2$$

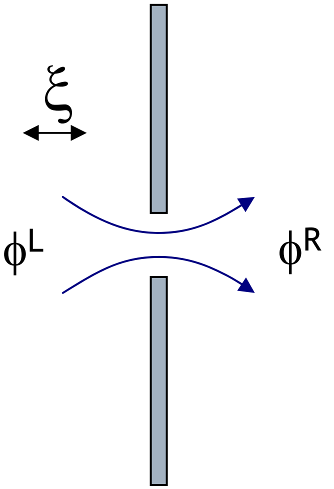
    at Josephson frequency

    $$\omega_{\mathrm{J}} \equiv \Delta\mu/\hbar$$
- Repeat for various temperatures ***T***
- Simple expectations
  - two regimes of ***T***
  - stronger signal at lower ***T***

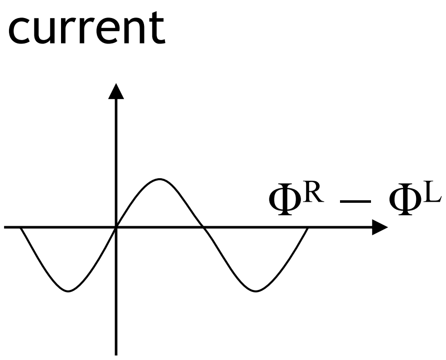


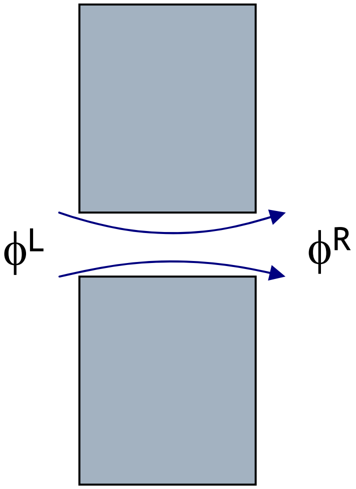


Aside: Superflow through a *single* aperture

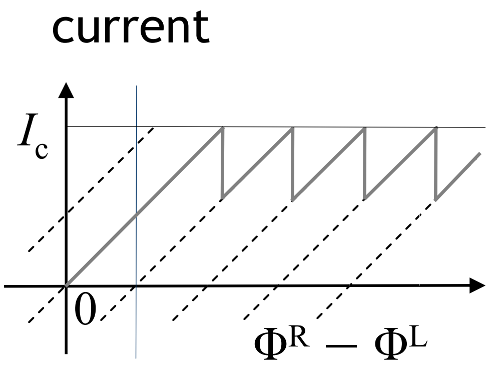


- Short aperture ($d<<\xi$)
  - Josephson current-phase relation $I \sim J \sin(\Phi^R - \Phi^L)$
  - single-valued current vs. phase
  - reversible phase slips (not dissipative)





- Long aperture ($d>>\xi$)
  - linear current-phase relation
  - multi-valued current vs. phase
  - characterized by *extrinsic* critical velocity (or equiv. critical twist)
  - dissipative phase slips

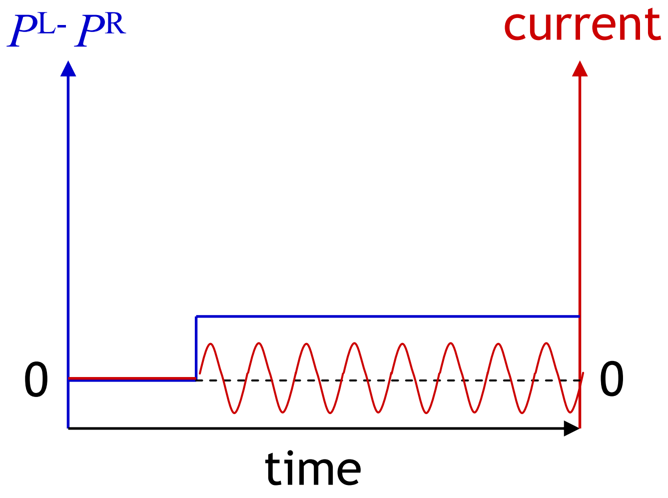


Expectations for the Berkeley group experiments II

- Basis for expectations
  - reservoirs described by phases: Φ^{L} & Φ^{R}
  - $\Delta\mu$ causes relative phase advance:

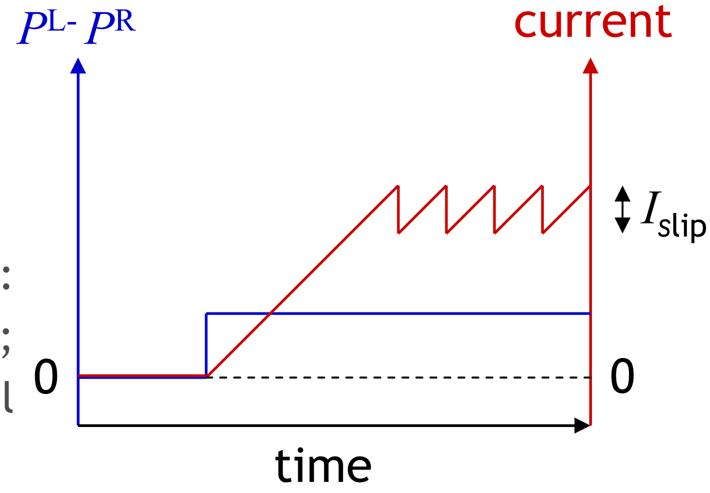
$$\frac{d}{dt}\left(\Phi^{\mathrm{R}}-\Phi^{\mathrm{L}}\right)=\frac{\Delta\mu}{\hbar}$$

  - builds up ‘twist’ & thus superflow through apertures
  - key ratio: healing length $\xi(T)$ to aperture length



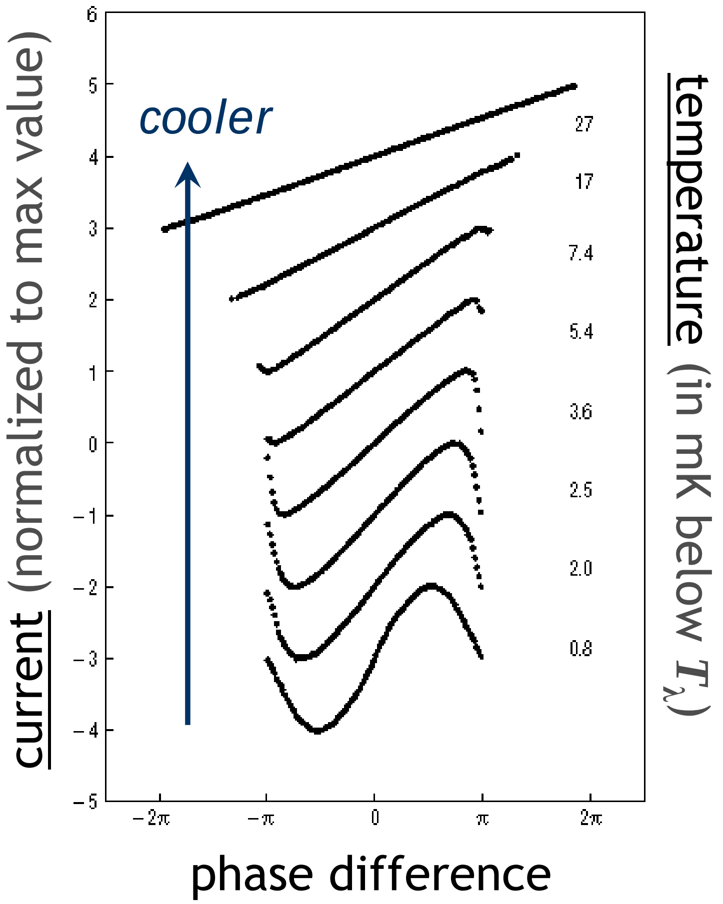
1. Higher T / shorter apertures:
reversible, no metastability;
less superfluid, weaker signal

2. Lower T / longer apertures:
irreversible phase slips, metastability;
more superfluid, stronger signal



What’s actually found by the Berkeley group I

- Current-phase relation evolving…
  - from sinusoidal Josephson behavior at high ***T***...
  - to linear (metastable, long ‘wire’ like) at lower ***T***

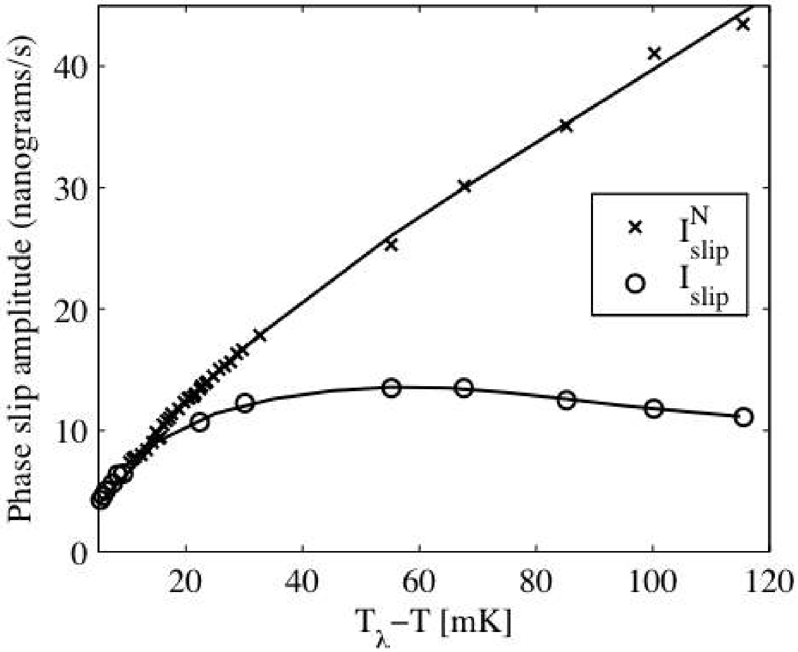


Hoskinson et al., *Nature Physics* **2**, 23 (2006)

What’s actually found by the Berkeley group II

- Examine all-apertures current power at Josephson frequency
- Repeat for lower & lower temperatures
  - apertures slipping synchronously?
    - expect *strengthening* signal
    - due to *increasing* superfluid density
  - instead broad, weak *maximum* is found?
  - suggests…
    increasing *lack of synchronicity*
    at lower temperatures

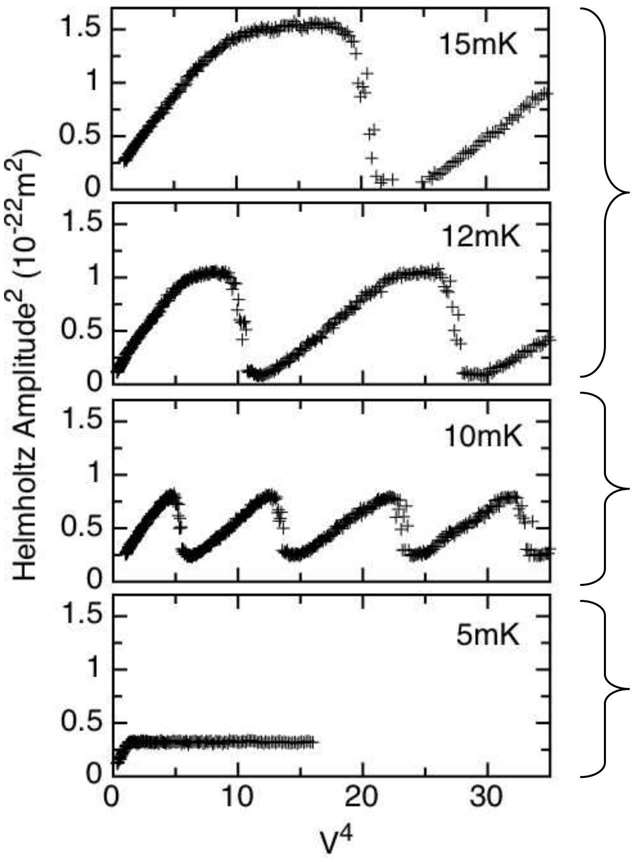
$I_{\text{slip}}(\omega_J)$ vs. T (experiment)



Sato et al., Phys Rev B 74, 144502 (2006)

What's actually found by the Berkeley group III

- Indirect observation: energy remaining in Helmholtz mode after phase-slippage stops



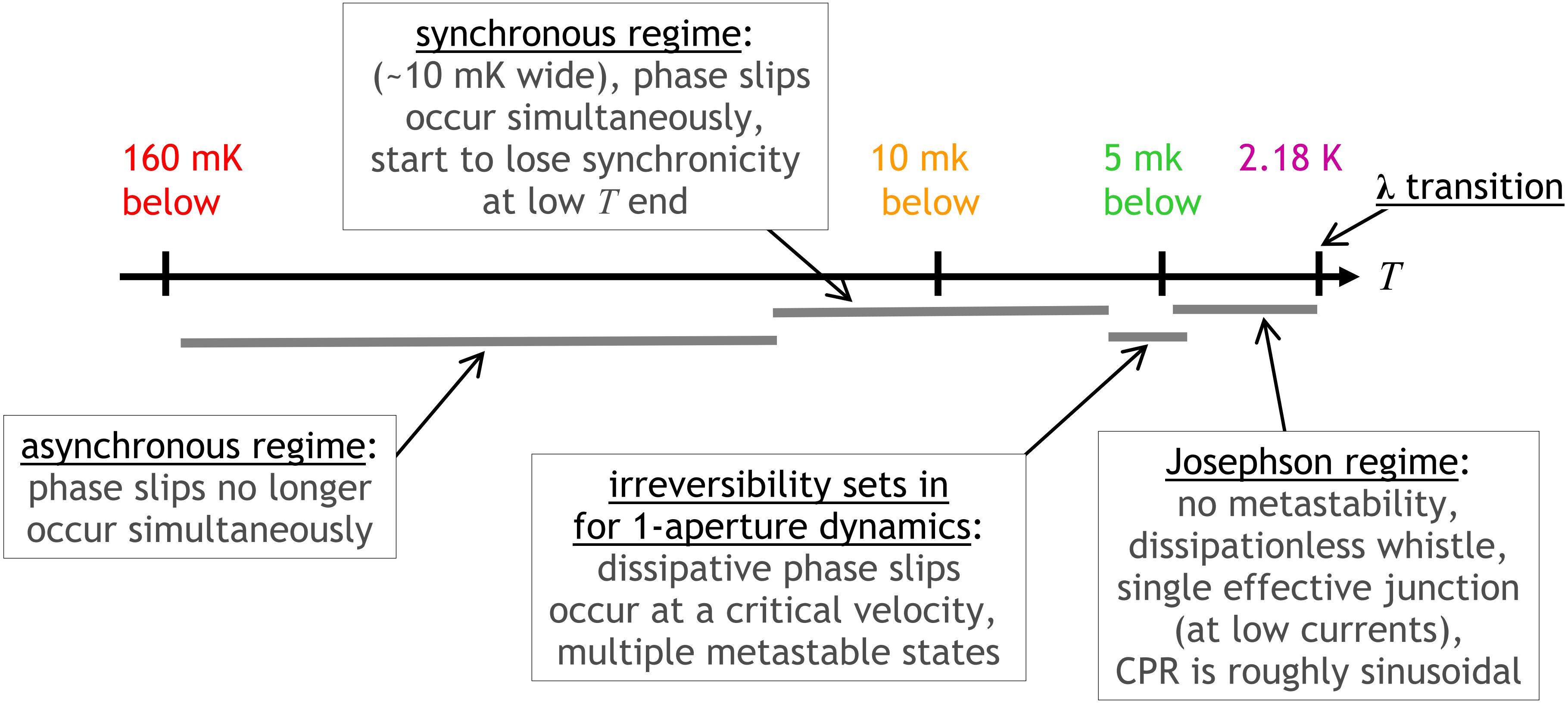
cooler still: indicative of non-simultaneous phase-slippage

cooler: indicative of simultaneous phase-slippage

cold: indicative of Josephson regime

Sato et al., PRB 74, 144502 (2006)

What features should one try to capture?



synchronous regime: (~10 mK wide), phase slips occur simultaneously, start to lose synchronicity at low T end

160 mK below

10 mk below

5 mk below

2.18 K

λ transition

T

asynchronous regime: phase slips no longer occur simultaneously

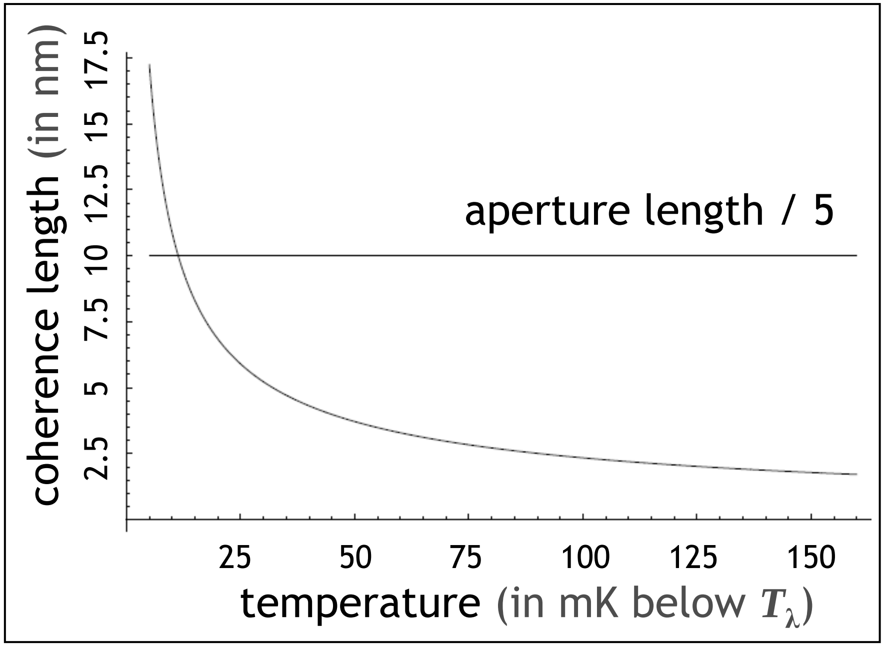
irreversibility sets in for 1-aperture dynamics: dissipative phase slips occur at a critical velocity, multiple metastable states

Josephson regime: no metastability, dissipationless whistle, single effective junction (at low currents), CPR is roughly sinusoidal

Back-of-the-envelope scenario I

Start with a *single* aperture

length~50 nm, dia~30 nm, ~cylindrical,



- **Examine coherence length $\xi(T)$**

$$\xi(T) \approx \xi_0 \left[1 - (T/T_\lambda)\right]^{-2/3}$$

$$\xi_0 \approx 0.3\,\mathrm{nm}$$

  - diverges at λ-point
  - microscopic at low T
- **Compare with aperture length / 5…**
  - smaller than ~ $\xi(T)$: Josephson regime
  - larger than ~ $\xi(T)$: phase-slip regime
- **Consistent with Josephson/phase-slip cross-over data**

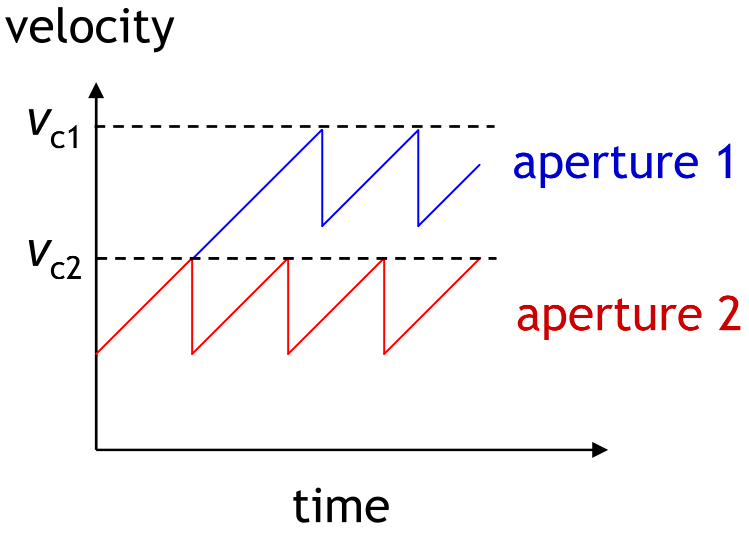
So for synchronous-to-asynchronous transition…

focus on phase-slip regime

Back-of-the-envelope scenario II

On to a *pair* of uncoupled apertures

- low T, so small ξ: phase-slip regime
- distinct critical velocities



- Asynchronicity of slip events
  - post-transient state
    - periodic: ap's slip at regular intervals
    - but mutually phase-shifted (τ_1, τ_2)
    - not 'maximal' net sawtooth
  - reduced power at Josephson frequency $|I(\omega_J)|^2 \sim 4\cos^2[\omega_J(\tau_1 - \tau_2)/2]$

Many (N) uncoupled apertures

- distribution of critical currents?
- power possibilities range between N^2 and N^1

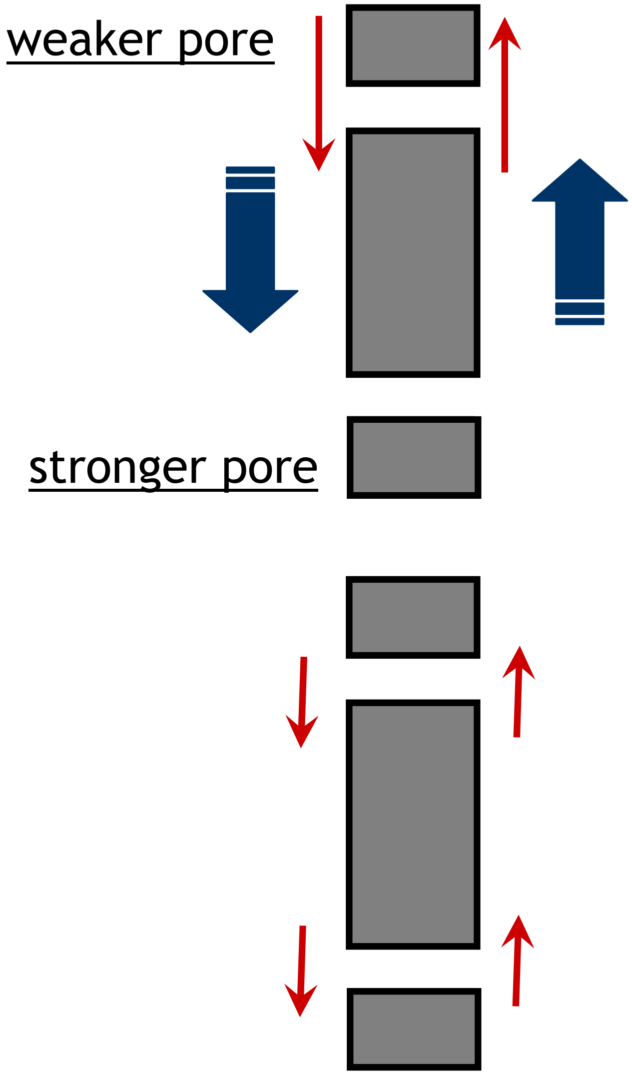
Back-of-the-envelope scenario III

Now for a pair of *coupled* apertures

- one weaker, one stronger

- Origin of coupling?
  - Δμ twists the phases in time
  - weaker aperture slips
  - it's less 'tense' so its phase advances
  - but this would sets up aperture-to-aperture reservoir flow
    - holds back weaker aperture's advance
    - advances stronger aperture
    - combats critical-current distinctions

- Impact of coupling?
  - promotes synchronicity
  - enhances power at Josephson frequency



Back-of-the-envelope scenario IV

Many apertures & critical-velocity heterogeneity (quenched? random?)

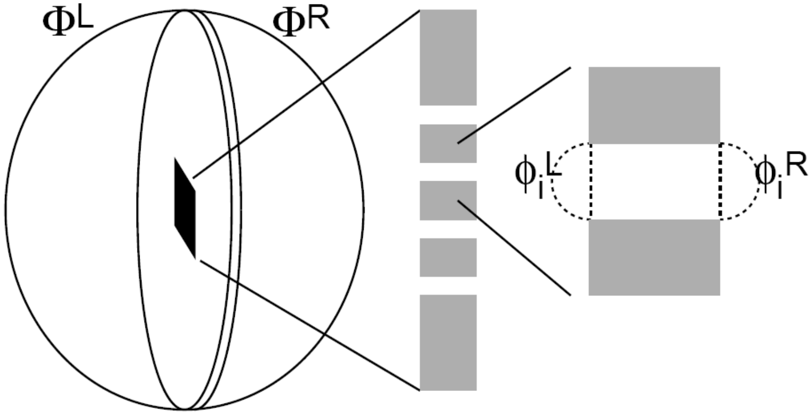
- Without aperture-aperture coupling?
  - ap’s slip asynch’sly during cycle
  - no system-wide avalanching
- With coupling?
  - combats heterogeneity & promotes synchronicity
- If heterogeneity beats coupling?
  - regime remains asynchronous
  - no system-wide avalanching
- If coupling beats heterogeneity?
  - generates synchronous regime
  - sys-wide avalanching: nonzero frac of ap’s slip synchrounously, despite heterogeneity

- Between the two?
  - a ‘non-equilibrium phase transition’

But… experimentally observed transition is temperature controlled?

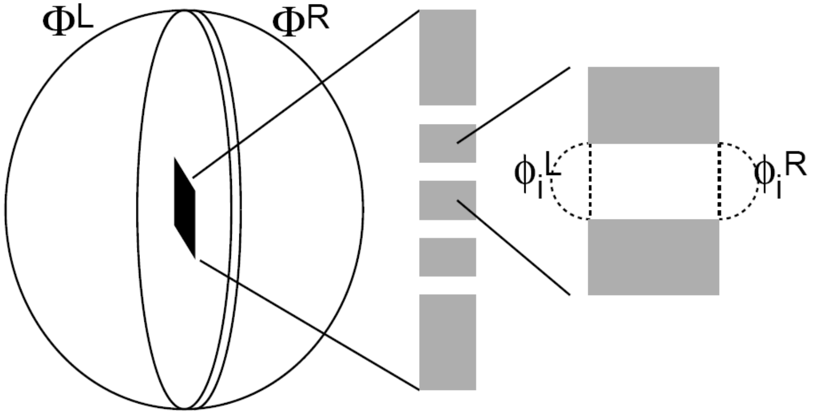
Elements of a theory I

- Two reservoirs
  - filled with superfluid ^{4}He
  - ignore spatial condensate-*amplitude* variations
  - allow spatial variations in *phase*
  - ‘zero T’: ignore thermal fluctuations



- Couple through nano-aperture array
  - also filled with superfluid ^{4}He
  - regular array (boundary conditions?)
  - apertures have identical geometries
  - but each characterized by a *random critical velocity* (i.e. a critical phase difference)

Elements of a theory II

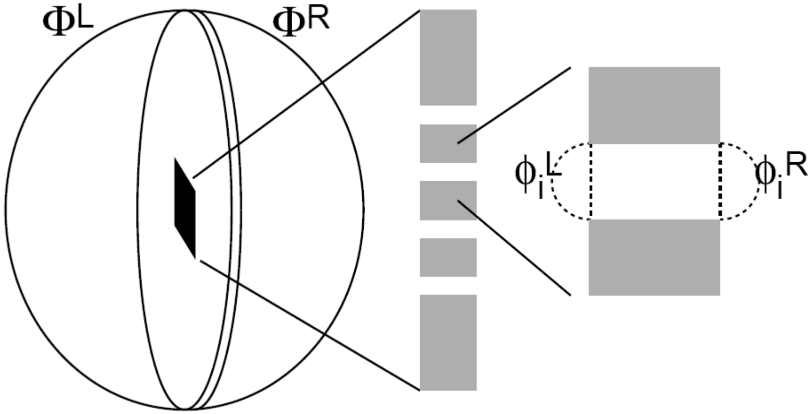


- Freedoms treated as controlled?
  - far-field phases (cf. Josephson-Anderson)
- Uncontrolled freedoms (‘spins’)?
  - phases near where reservoirs meet apertures
  - phase slipped by each aperture
- State & dynamics?
  - not thermal equilibrium
  - system quasi-statically follows local energy minimum
  - punctuated by phase slips, occurring deterministically

Elements of a theory III

- Energetics?
  - flow kinetic energy in reservoirs L & R ('spins' & controls)

$$H^{\mathrm{L/R}} = \tfrac{1}{2} K_s \int_{\mathrm{L/R}} d^3 r \left| \boldsymbol{\nabla} \chi^{\mathrm{L/R}}(\mathbf{r}) \right|^2$$



  - & flow kinetic energy in each aperture i ('spins' & phase slips for each)

$$H_i = \tfrac{1}{2} K_s J \left(\phi_i^{\mathrm{L}} - \phi_i^{\mathrm{R}} - 2\pi n_i \right)^2$$

  - plus matching/boundary conditions at aperture ends & control phases
  - & a quenched random critical velocity (ie 'twist') for each aperture

Elements of a theory IV

- Eliminate reservoir phase fields
  - invoke Anderson’s electrostatics analogy (RMP ’66)
  - arrive at ‘spins & slips & controls’ Hamiltonian

$$E = \frac{K_s}{4} \sum_{ij} (\phi_i^{\mathrm{L}} - \Phi^{\mathrm{L}})\, C_{ij}\, (\phi_j^{\mathrm{L}} - \Phi^{\mathrm{L}}) + (L \to R) + \frac{JK_s}{2} \sum_i \left(\phi_i^{\mathrm{R}} - \phi_i^{\mathrm{L}} - 2\pi n_i\right)^2$$

effective inter-aperture interaction mediated by bulk superfluid

energy inside apertures

- ‘capacitance’ coupling matrix, long-ranged

$$\left(C^{-1}\right)_{ij} = \frac{\delta_{ij}}{4\pi\, r_0} + \frac{1 - \delta_{ij}}{4\pi\, |r_{ij}|}$$

aperture diameter

aperture separation

How does this model behave? I

- Numerics

  1. fix $\Phi^R - \Phi^L$.
  2. find $\{ \phi^L_i , \phi^R_i \}$ by energy minimization.
  3. increment n_i in apertures in which current exceeds critical.
  4. go to step 2 until no new phase-slips are found in step 3.
  5. increment 'time' (i.e. increment $\Phi^R - \Phi^L$). go to step 2.

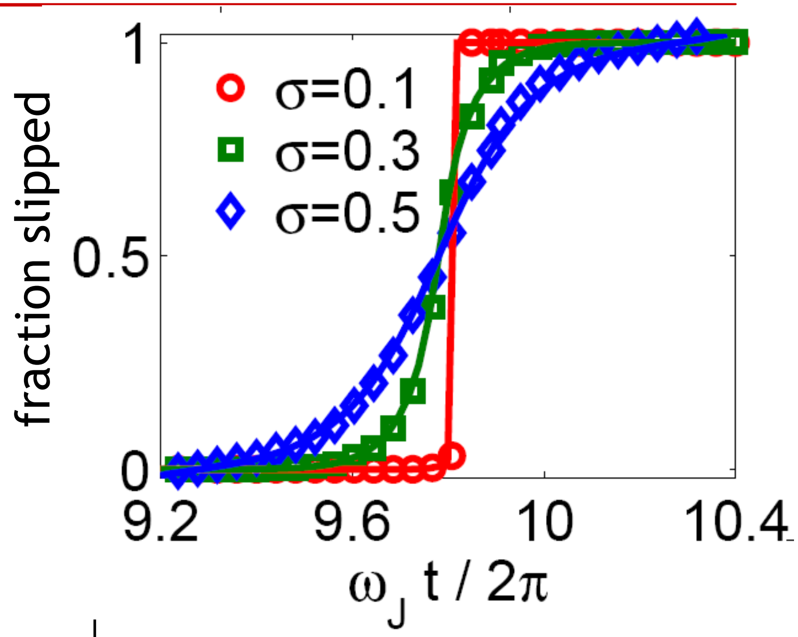
- Analytics: so far, a kind of mean-field theory
  - similar to ones used in prior work on CDWs, random magnets…

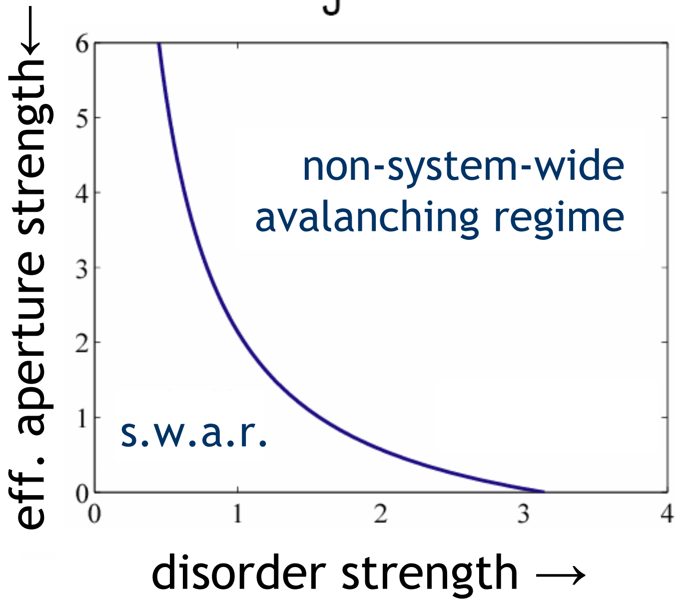
- What emerges?

How does this model behave? II

- Fraction slipped vs. ‘time’ through the cycle
  - low disorder: jumps
    system-wide avalanche
    some synchronicity
  - high disorder: glides
    no system-wide avalanche
    no synchronicity
  - critical disorder line,
    nonequilibrium phase transition

(lines: MFT; points: numerics)



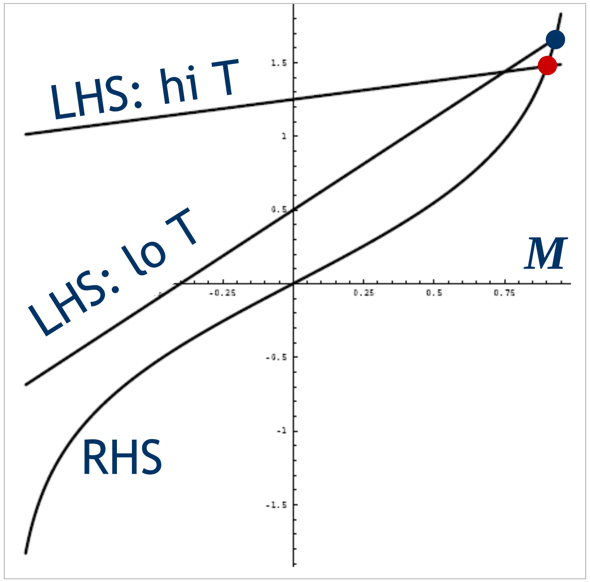


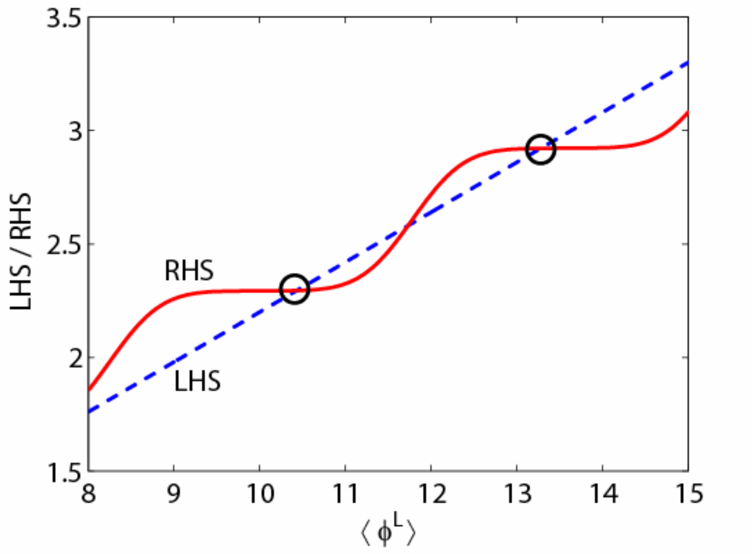
Aside: caricature of MFT

- MFT mechanism for ferromagnetism

  $$(H/T) + (J/T)M = \operatorname{arctanh} M$$

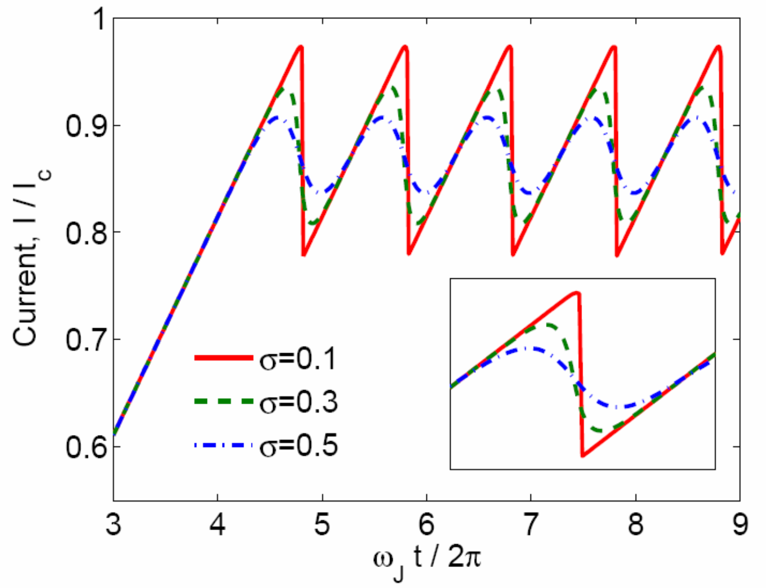
  - plot LHS & RHS vs. M
  - y-int H/T, slope J/T
  - how does the solution evolve with H/T at fixed J/T ?
- Similar mech for avalanching systems
  - steep slope = high disorder: *continuous* evolution, no system-wide avalanches
  - shallow slope = low disorder: *discontinuous* evolution, system-wide avalanches
  - standard mechanism from several earlier settings (RFIM, CDW,…)



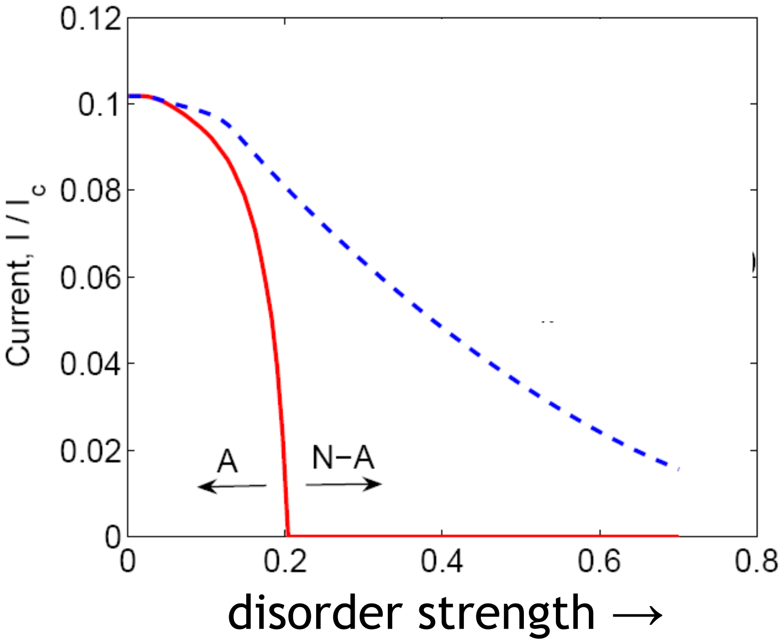


How does this model behave? III

- Impact on current vs. ‘time’
  - zero disorder: strong sawtooth
  - small disorder: drop shrunk, sawtooth rounded
  - larger disorder: drop washed out



- Impact on current vs. disorder
  - current drop ~ an ‘order parameter’
  - but current oscillation amplitude remains into the non-s.w.a. regime



So where do we stand?

- Seem to capture some relevant phenomenology
  - disorder vs. coupling competition
  - triggers system-wide avalanching transition
  - synchronicity diminished with…
    - increasing disorder
    - reduced coupling

- But…
  - experiments see synchronicity loss *with reduced* ***T***
  - theory sees synchronicity loss *with increased disorder* at zero ***T***

One more ingredient

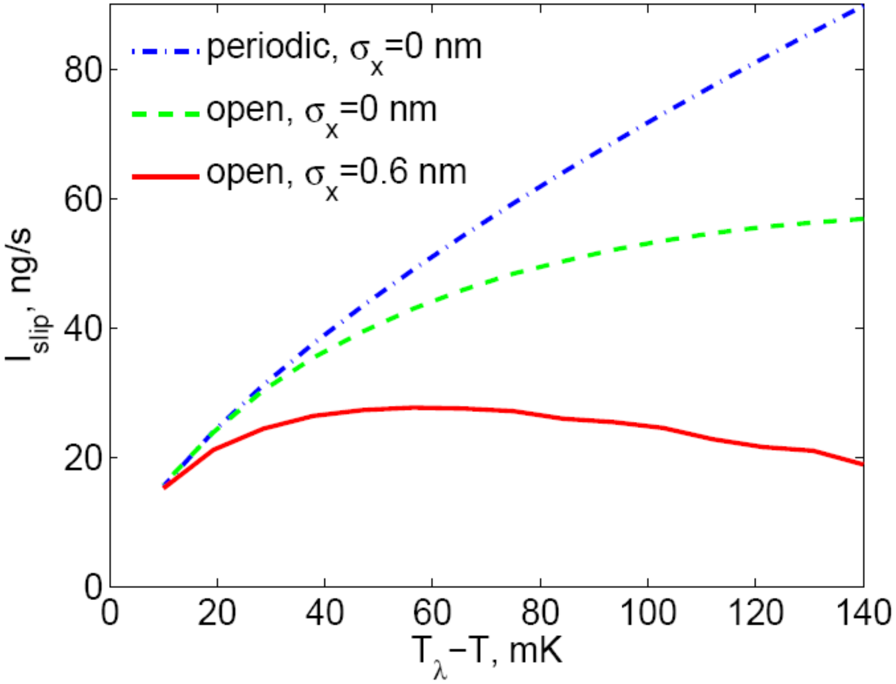
- Perhaps temperature effectively tunes disorder?

- Plausibility argument: as ***T*** is reduced…
  - superfluid healing length shrinks (from macro to micro)
  - less able to heal variations/imperfections (e.g. aperture surface roughness)
  - some healed disorder effectively ***resurges***

- Simple model
  - assume critical velocity near T_λ: $v_{i,c}(T) \approx \hbar/m\xi(T)$ for aperture i
  - modify for local randomness: $v_{i,c}(T) \approx \hbar/m[\xi(T) + x_i]$ (take ‘defect’ size x_i Gaussian)
  - smaller $\xi \rightarrow$ larger impact from random x_i

‘Disorder-driven’ phenomenology via temperature

- Impacts of reducing temperature
  - increase superfluid density
  - weaken healing of disorder
- Shows in current oscillations
  - blue: no disorder
  - red: with disorder
    - reminiscent of experiments

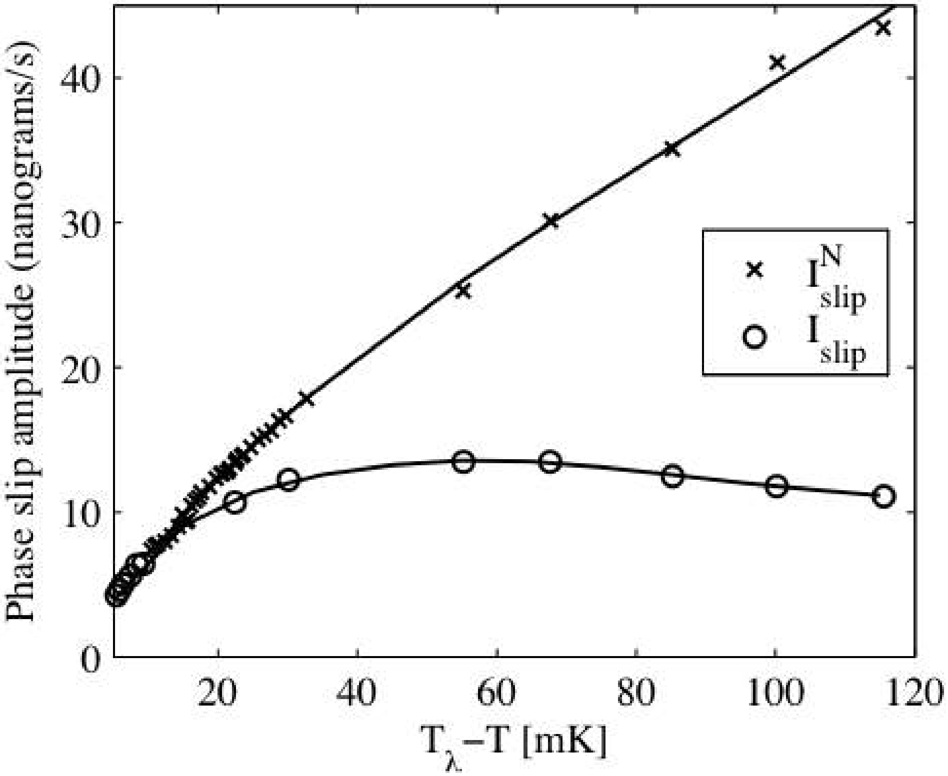


- New issue: *deterministic* heterogeneity
  - boundaries also create heterogeneity via impact on flow pattern
  - also produces asynchronicity, even with no disorder (green)
  - funnels, inlets, engineering of the array housing?

What’s actually found by the Berkeley group

- Examine all-apertures current power at Josephson frequency
- Repeat for lower & lower temperatures

$I_{\text{slip}}(\omega_J)$ vs. T (experiment)



Sato et al., Phys Rev B 74, 144502 (2006)

Concluding remarks

- Transition between system-wide avalanching regime & non-s.w.a.r?
    - real? direct observations?
    - avalanche size statistics & scaling; beyond MFT
    - spatial structure, physical dynamics
- Role of aperture array geometry?
    - lattice size, shape, topology; interaction range
    - stochastic vs. deterministic heterogeneity
    - funnel engineering?
- Role of thermal fluctuations?
- Potentially useful setting for noneq. phase transitions
    - complements others (random-field magnets, earthquakes, CDWs…)

Concluding remarks

> “Any experiment you can
> do in condensed matter,
> you can do better in helium”

(C.C. Grimes → D.S. Fisher → PMG)

Acknowledgements

- UIUC colleagues
  Gordon Baym, Karin Dahmen,
  Smitha Vishveshwara & Tzu-Chieh Wei

- U.S. National Science Foundation
  & U.S. Department of Energy

- Richard Packard, Yuki Sato & Aditya Joshi

- Pekker, Barankov & PMG, *cond-mat/0606560*