Superconductivity at the Nanoscale

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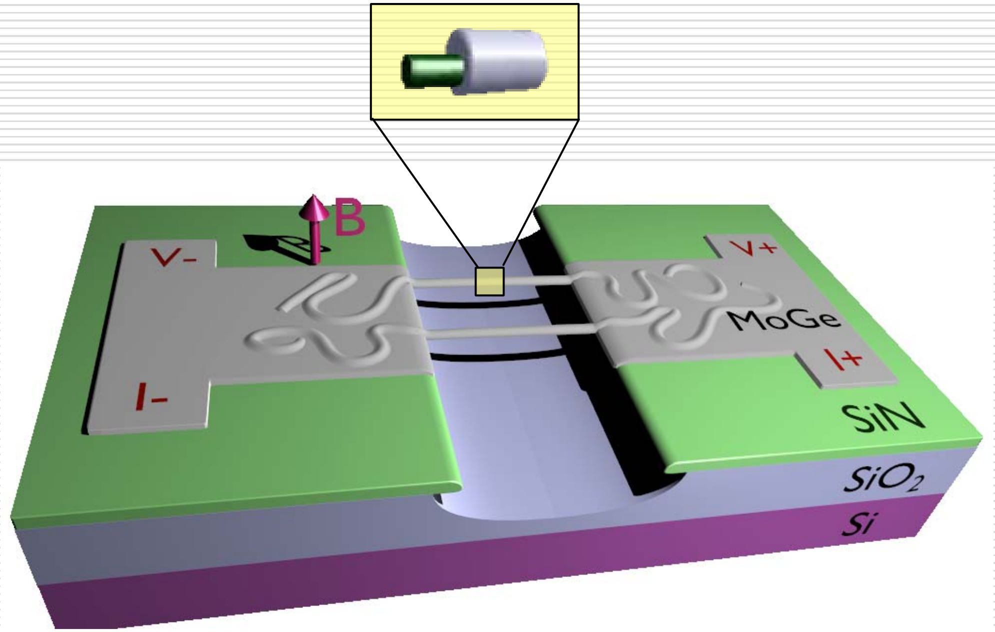


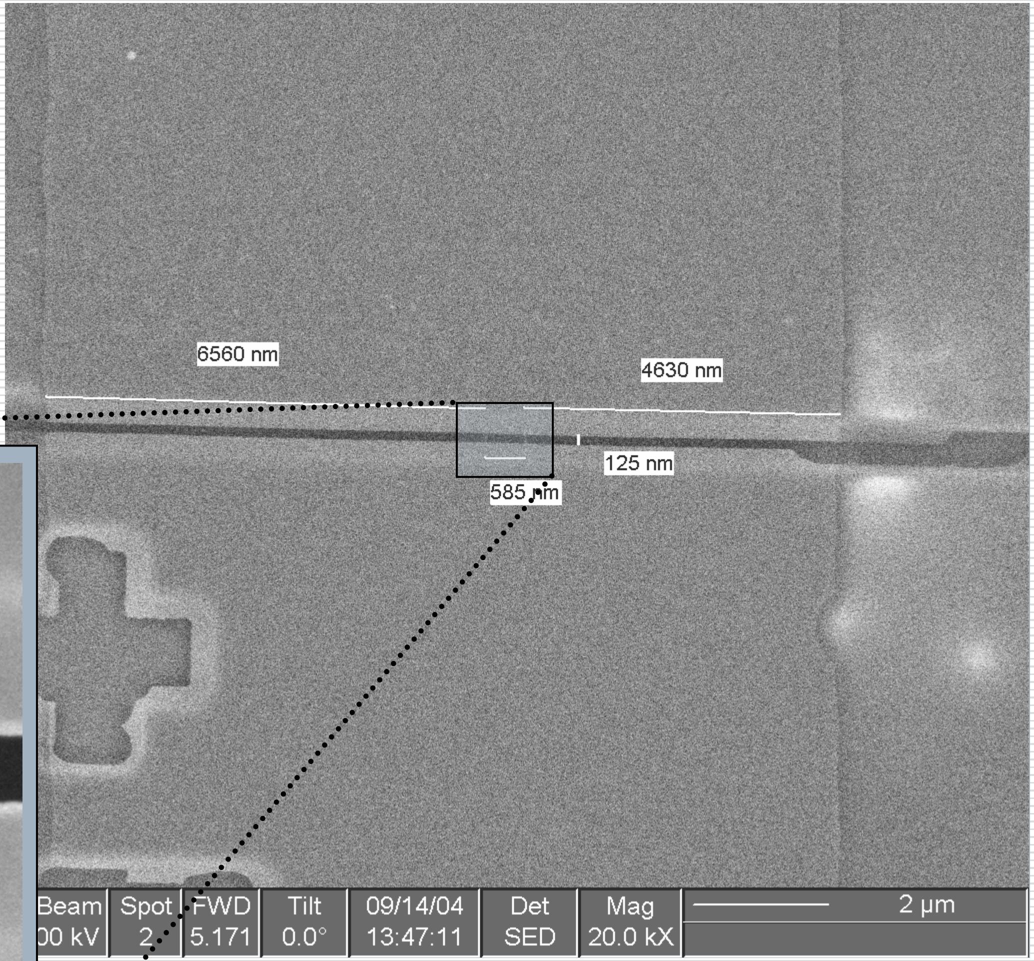
Talk outline

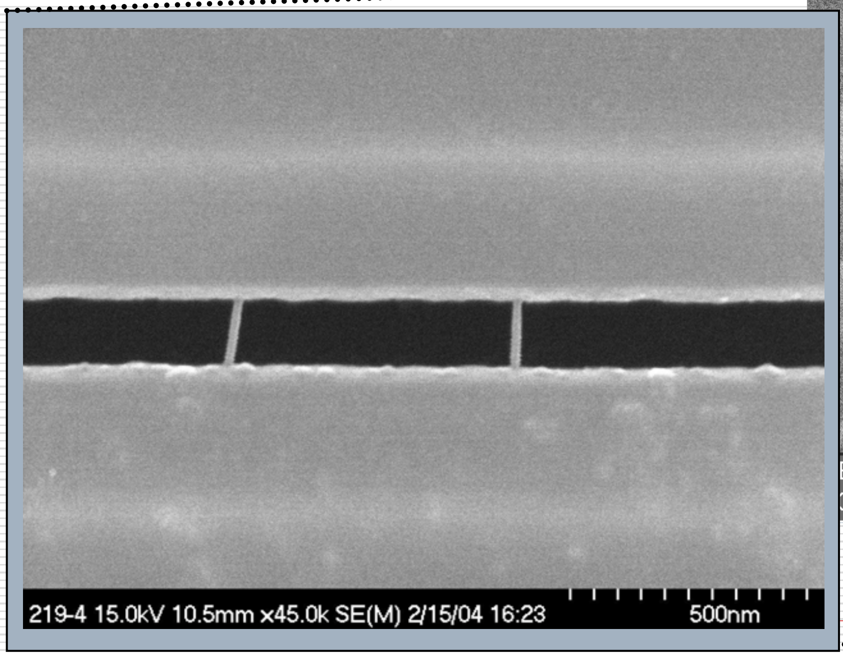
- Elements of a nanoscale SQUID
- Fabrication, DNA templating
- Measuring the resistance

  vs. magnetic field & temperature
- Origins of resistive behaviour
- Back-of-the-envelope picture
- Some comparisons with experiments
- A simple model: fluctuating Josephson junctions
- Refinements: LAMH order parameter fluctuations
- A superconducting phase gradiometer?
- Concluding remarks

Elements of a nanoscale SQUID

- Leads: thin films of MoGe type II superconductor
- Separated by 125nm trough
- DNA molecules as bridge templates
- Sputter MoGe (~9nm thick films)
- Bridge x-section (~5nm thick x 20nm wide)
- Typical bridge resistance ~3kΩ



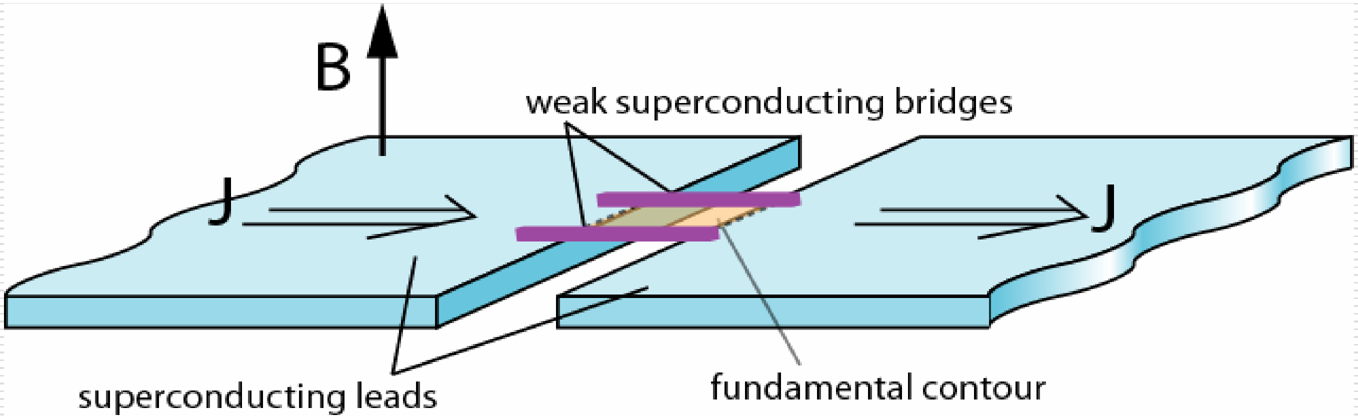


- *Hopkins/Bezryadin devices (2004)*

Schematic of a nanoscale SQUID

- Thin-film s/c leads
- Connected by s/c bridges
- Makes a superconducting quantum interference device
- Measure device resistance vs.

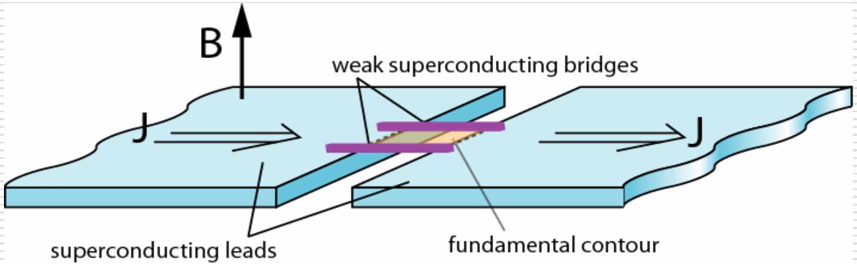
  magnetic field, temperature, current,…

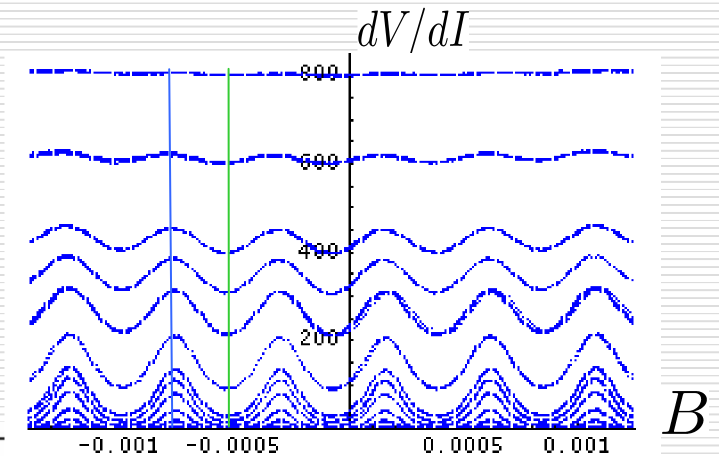


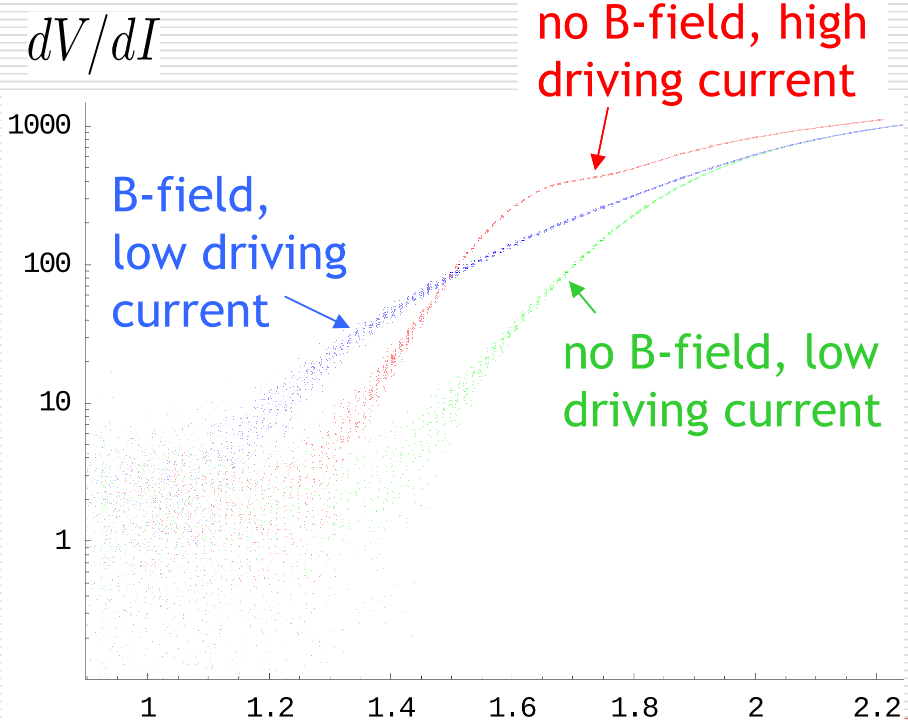
What do resistance measurements yield?

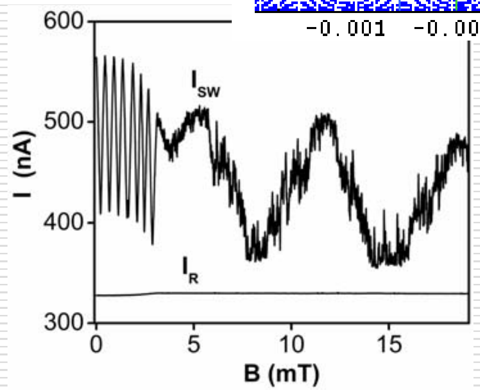
- Scale?
- Field regimes? Oscillations?
- Temperature dependence?

Hopkins & Bezryadin data (2004)









Some issues to be addressed

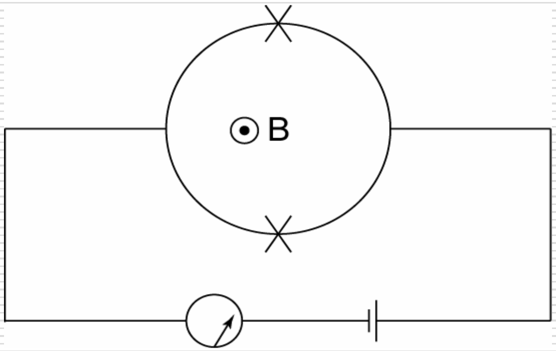
Device resistance…

- origin, scale
- dependence on…
  - *temperature*
  - *magnetic field*
  - *current*

Oscillations with field…

- origin
- regimes
- periods
- amplitudes

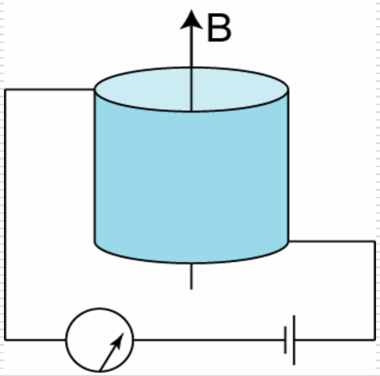
Related phenomenology: Flux dependence

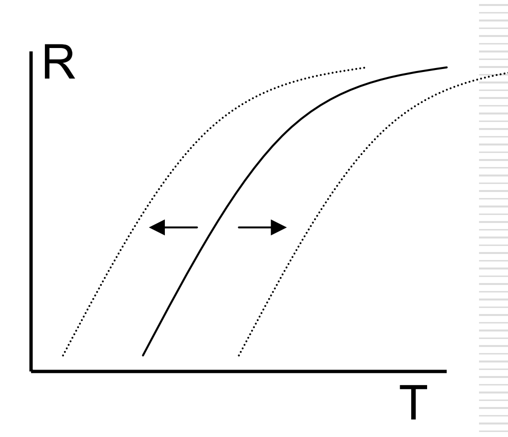


dc SQUID

- *superconducting loop*
- *pair of Josephson junctions*
- *flux-threaded*

• critical current I_c? oscillates with field, period Φ_0/area





Little-Parks effect

- *superconducting cylinder*
- *thin-walled*
- *flux-threaded*

• resistance near T_c?

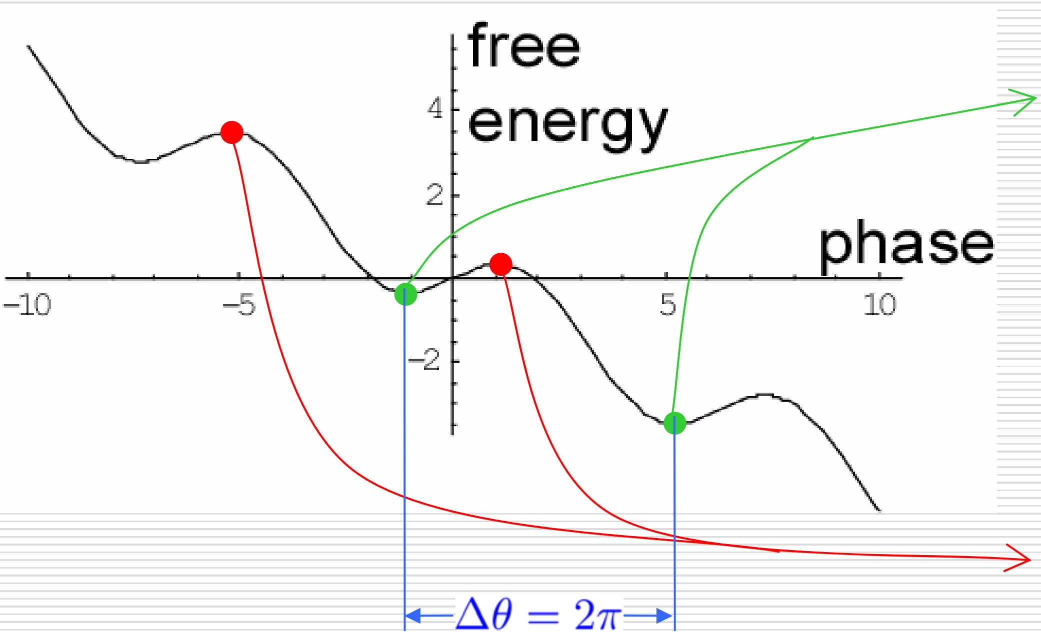
- oscillates with field, period Φ_0 /area

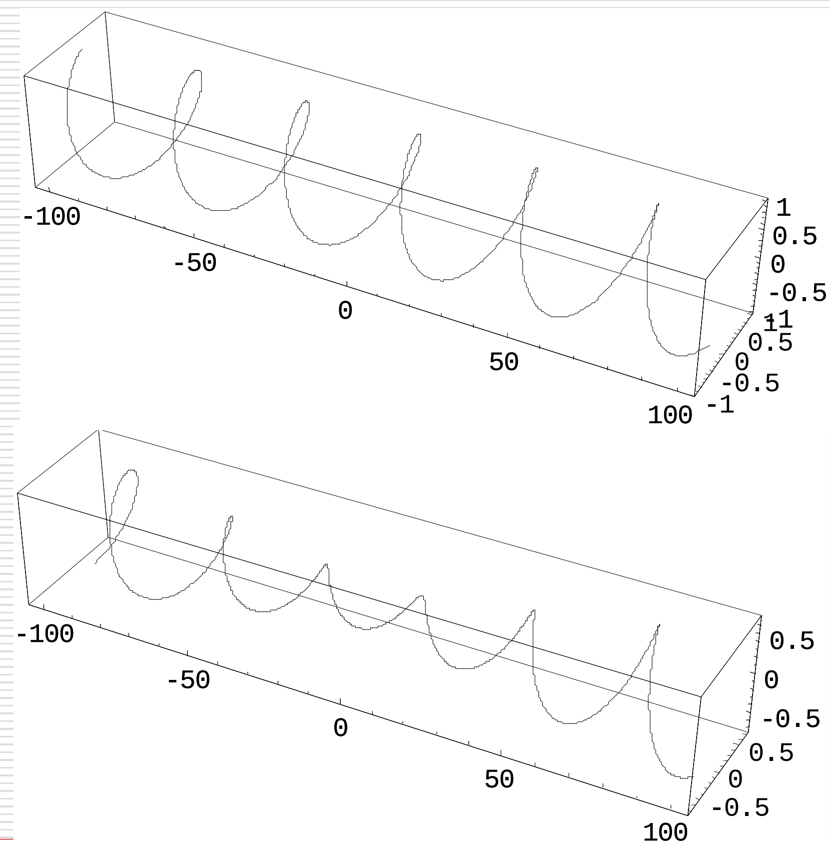
Related phenomenology: Intrinsic resistance

Little, Phys. Rev. **156**, 368 (1967)
Langer & Ambegaokar, Phys. Rev. **164**, 498 (1967)
McCumber & Halperin, Phys. Rev. B **1**, 1054 (1970)
McCumber, Phys. Rev. **172**, 427 (1968)

Single thin superconducting wire

- *bias current: preferred twist*
- *phase slips: activated thermally*
- *net rate: average voltage, effective resistance*





Metastable & transition states are stationary points of free energy

Related phenomenology: Intrinsic resistance

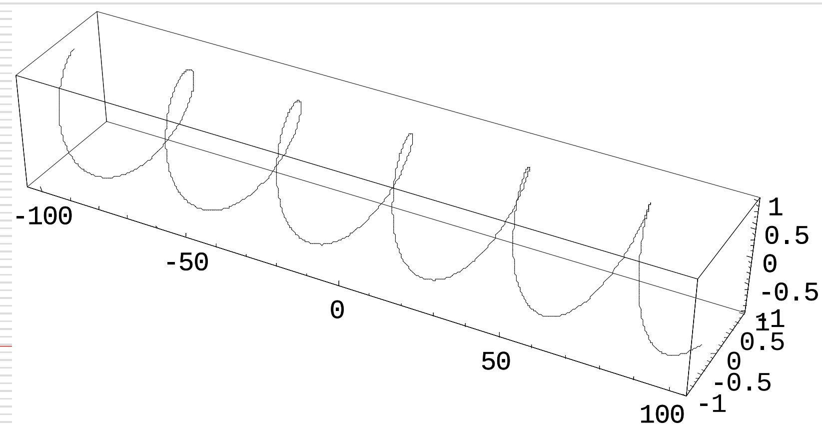
Single thin superconducting wire

- scheme: stochastic (Langevin/Fokker-Planck/Arrhenius)
- result (LA-MH): *voltage vs. current & temperature*

$$V(I,T) = (\hbar\Omega/e)\exp\left(-\Delta F/kT\right) \times \sinh\left(\pi\hbar I/2ekT\right)$$

- *voltage* V
- *current* I
- *temperature* T
- *energy barrier* ΔF,
  $\Delta F \sim H_c^2 \xi A$
- *coherence length* ξ,
  area A, *critical field* H_c
- *attempt frequency* Ω

...Ohmic at low currents

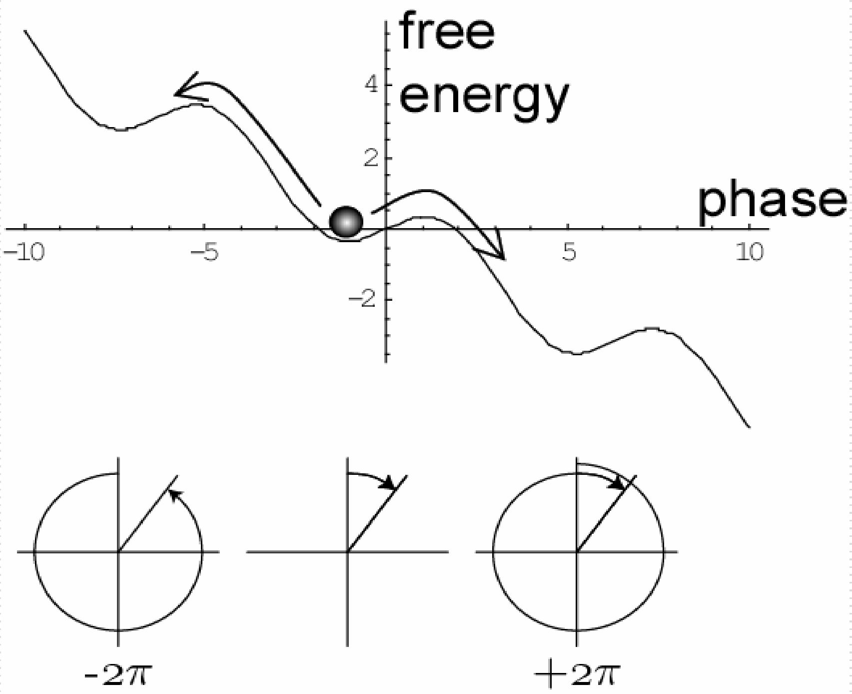


Related phenomenology: Intrinsic resistance

Ivanchenko & Zil'berman, JETP Lett. **8**, 113 (1968)
Ivanchenko & Zil'berman, JETP **28**, 1272 (1969)
Ambegaokar & Halperin, PRL **22**, 1364 (1969)

Single Josephson junction

- bias current: *preferred phase*
- phase slips: *activated thermally*
- balance: *net voltage, effective resistance*



$$G(I,\Theta) = -\frac{\hbar}{2e}\left(I_c \cos\Theta + I\,\Theta\right)$$

- Langevin/Fokker-Planck scheme

$$\dot{\Theta} = -\tfrac{1}{2}\partial_{\Theta} G + \nu(t)$$

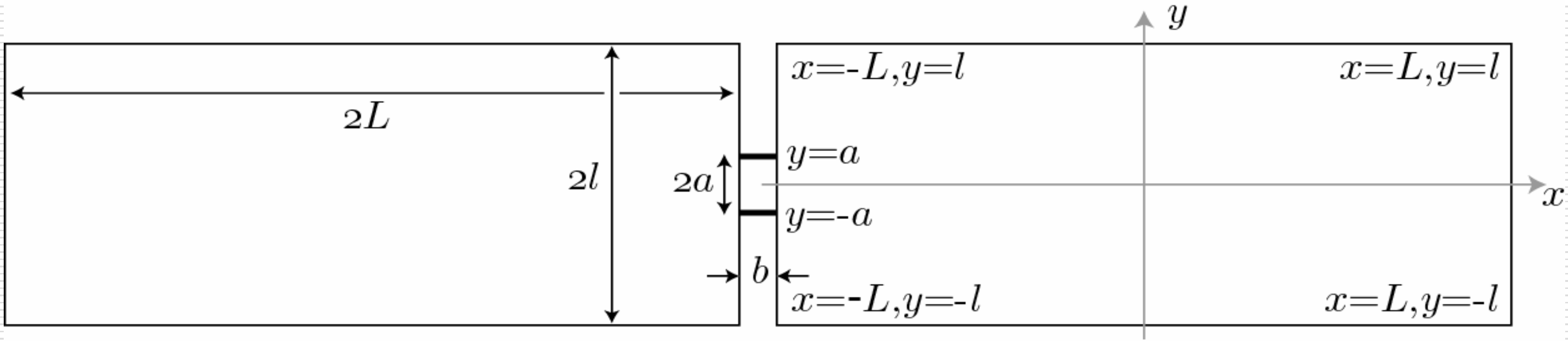
- Rate of TAPS (thermally activated phase slips)

$$\dot{\Theta} \sim 2\pi\,(\text{forward} - \text{reverse})$$

- Josephson relation: voltage

$$V \sim \dot{\Theta}$$

Rough length-scales of H-B device



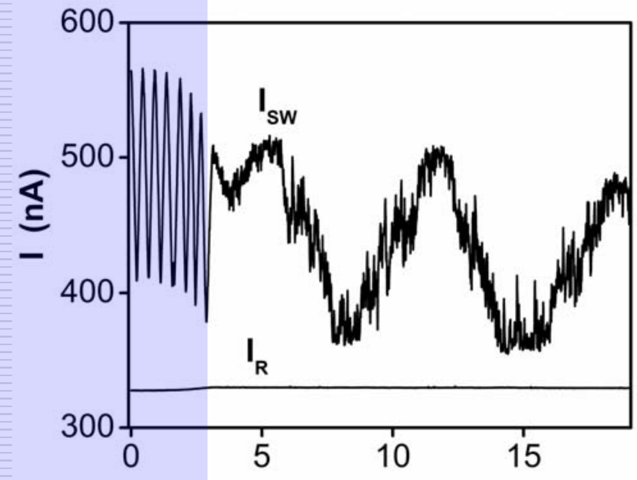
penetration depth λ	$100\ \mu m$
lead width $2l$ (meso. length)	$13\ \mu m$
coherence length ξ	$10\ nm$
lead length $2L$	large (mm)
bridge separation $2a$	$500\ nm$
trench width b	$130\ nm$

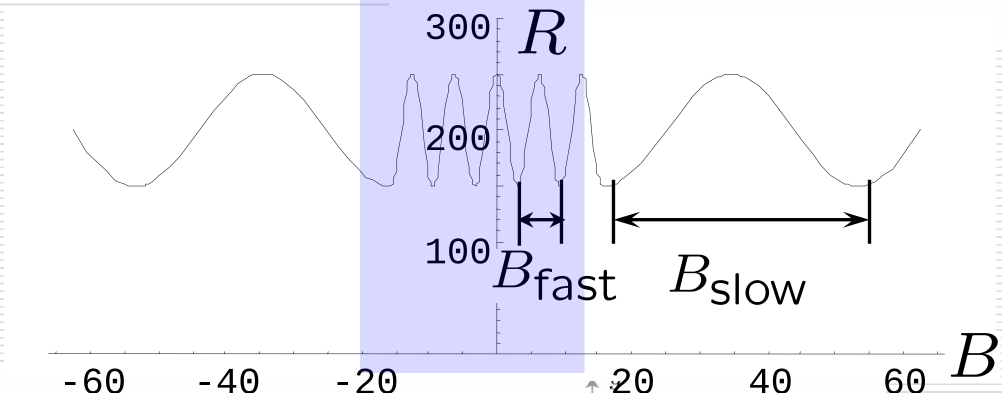
Field regimes: What's observed?

low B: fast osc's

period $B_{\text{fast}} \approx \frac{1}{10} B_{\text{slow}}$

x-over scale ~2 mT

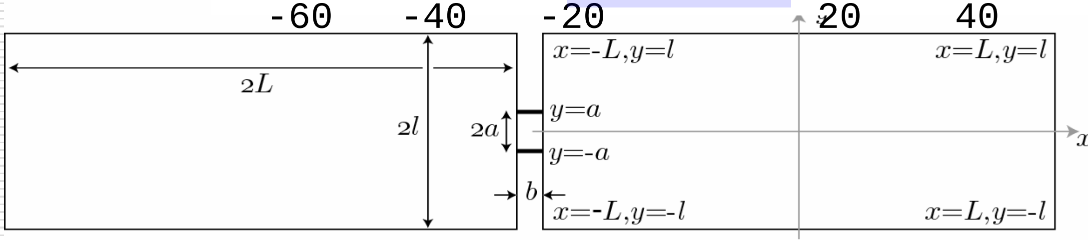




high B: slow osc's

period $B_{\text{slow}} \approx \frac{\Phi_0}{2ab}$

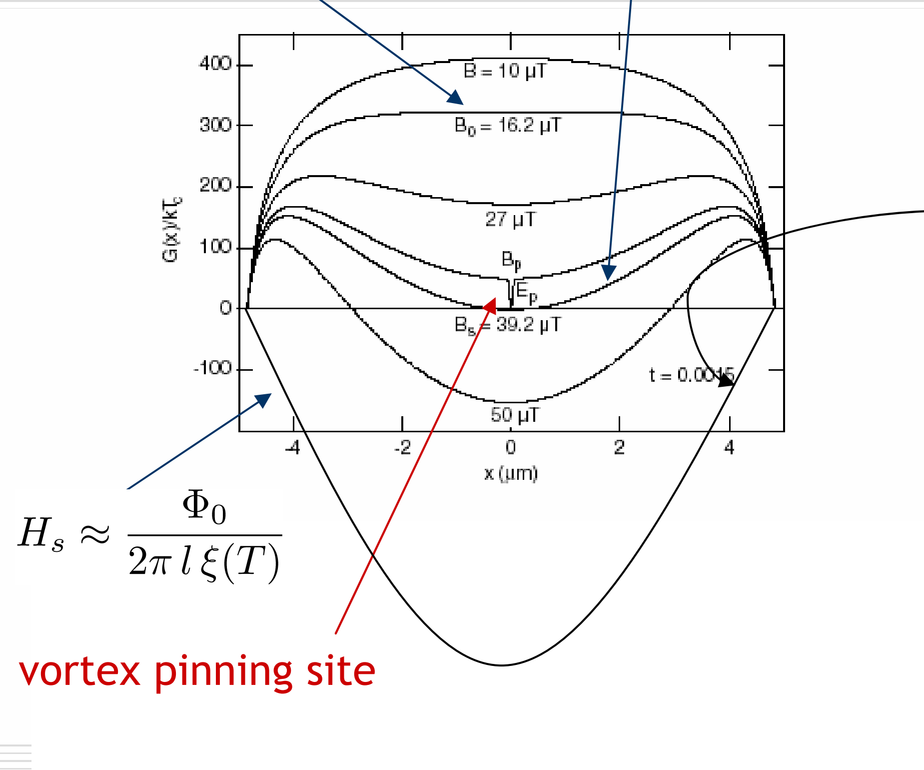
(Aharonov-Bohm: cf. Little-Parks, SQUID,...)



Field regimes: Likharev's critical fields

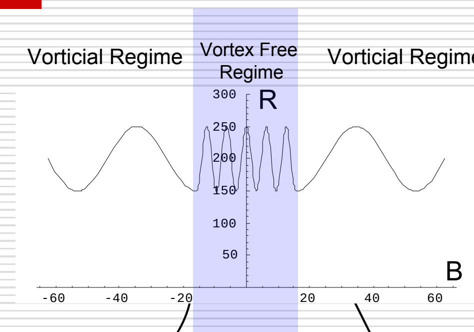
$$H_{c0} = \pi\Phi_0/l^2$$

$$H_{c1} = \frac{\Phi_0}{2\pi l^2}\ln\left(\frac{l}{2\xi(T)}\right)$$



$$H_s \approx \frac{\Phi_0}{2\pi\, l\, \xi(T)}$$

vortex pinning site



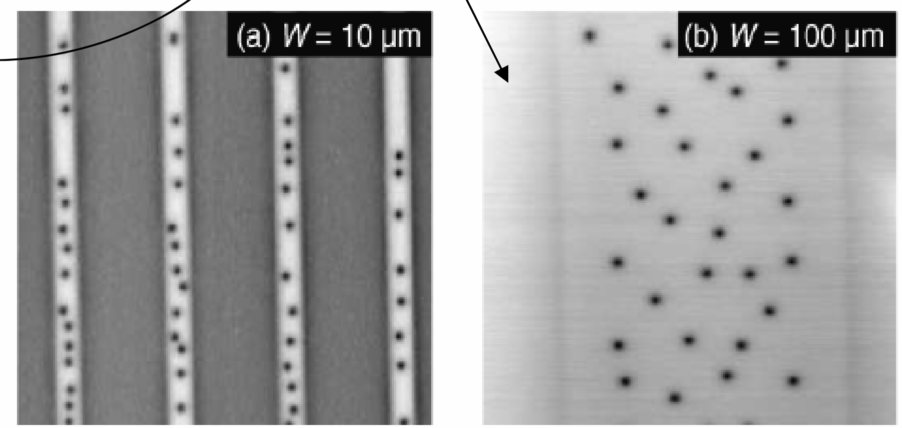
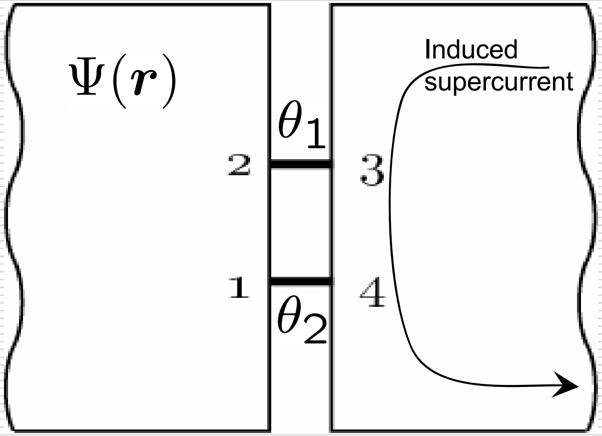


FIG. 2. (a) 10 μm strip after field cooling in 85 μT. The strips appear light because of the Meissner expulsion of the field, but many vortices (darker spots) are visible. (b) 100 μm strip after field cooling in 5.3 μT. Both images are 140 μT full scale, and about 145 μm wide.

Likharev, Sov. Radiophys. **14**, 722 (1972)
Stan, Field & Martinis, PRL **92**, 097003 (2004)

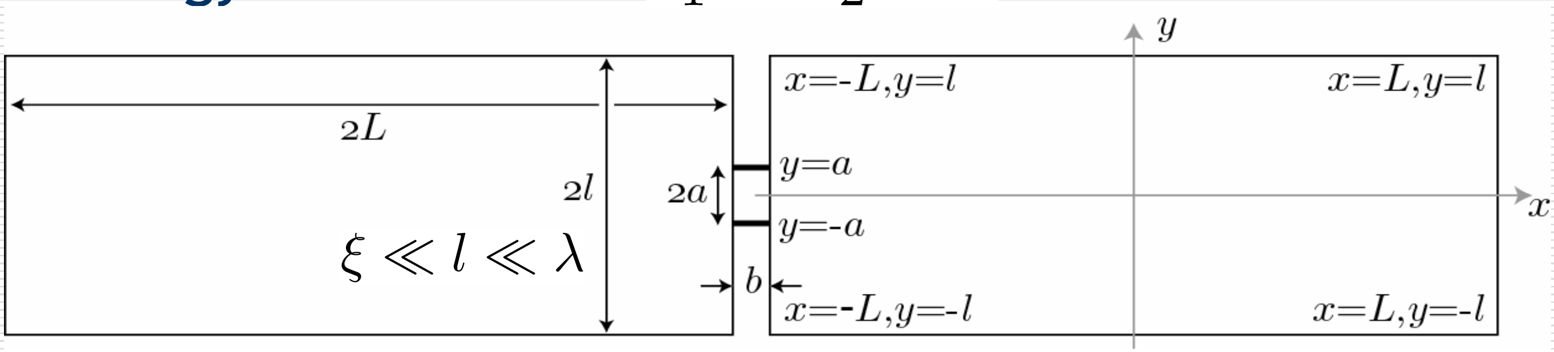
Low-field regime: Simple picture of periodicity



- **Leads**
  - mesoscopic
  - well penetrated by field
    $\Psi(\boldsymbol{r}) = \Psi_0 \exp i\phi(\boldsymbol{r})$
  - simple phase profile $\phi(\boldsymbol{r})$
  - simple current pattern

$$E \sim -(\cos\theta_1 + \cos\theta_2)$$

- **Bridges**
  - model as Josephson junctions, weak feedback on leads
  - energy minimum at $\theta_1 = \theta_2 = 0$



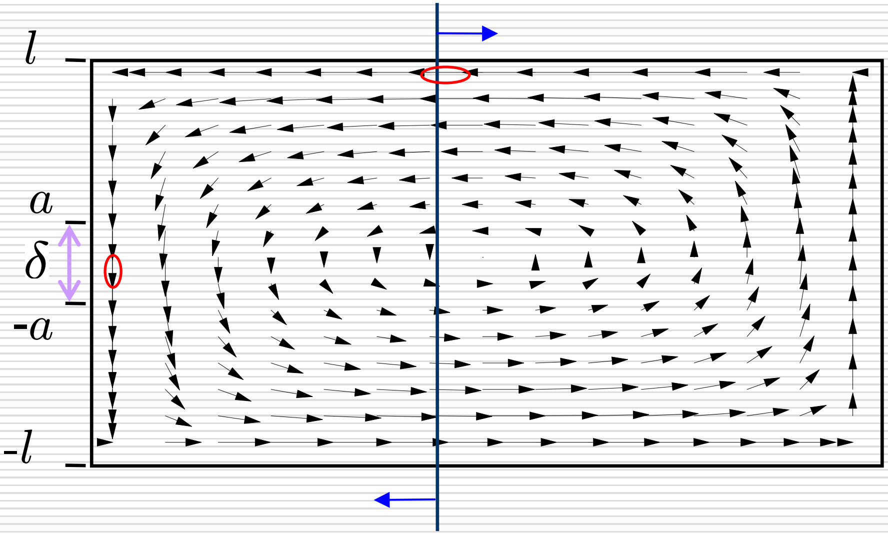
Low-field regime: Simple picture of periodicity

- Vector potential (gauge choice)

$$\boldsymbol{A}(\boldsymbol{r}) = By\boldsymbol{e}_x$$

- Top/center of strip: London gauge

$$\boldsymbol{j}(y) = -\frac{4e^2 n_s}{mc}\boldsymbol{A}(y) = -\frac{4e^2 n_s}{mc}By\boldsymbol{e}_x$$



- Current at edge $\boldsymbol{j} \approx -\frac{4e^2 n_s}{mc}Bl\boldsymbol{e}_x$

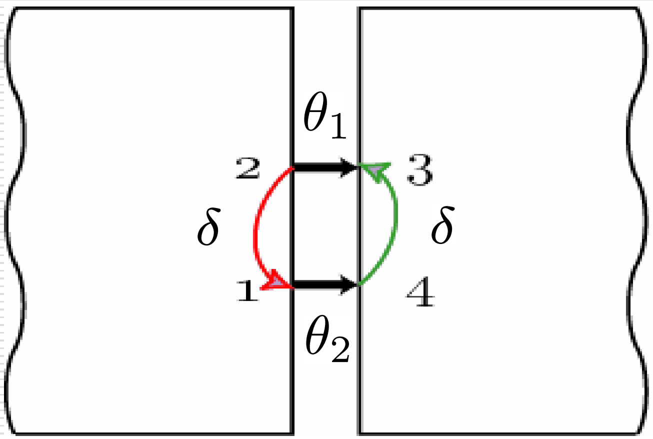
- Phase gradient on edges

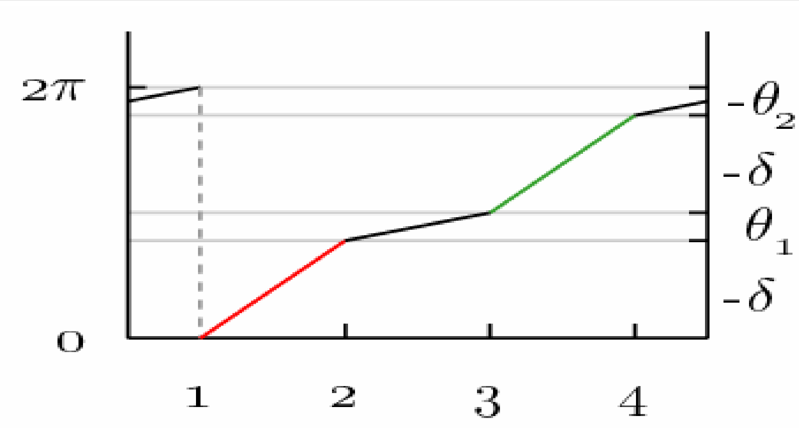
$$\boldsymbol{j} = \frac{2e\hbar n_s}{m}\boldsymbol{\nabla}\phi$$ (GL theory), so $$\boldsymbol{\nabla}\phi = \frac{m}{2e\hbar n_s}\boldsymbol{j} = \frac{2e}{\hbar c}Bl\boldsymbol{e}_x$$

- Phase gain between bridges $\delta = \int_{-a}^{a} \boldsymbol{\nabla}\phi \cdot d\boldsymbol{r} = \int_{-a}^{a} \frac{2e}{\hbar c}Bl\, dy \approx \frac{2e}{\hbar c}B\,2al$

Low-field regime: Simple picture of periodicity

- Phase gain between bridges $\delta \approx (2e/\hbar c) B\, 2al$





- Phase constraint

$$\theta_1 - \delta - \theta_2 - \delta = 2\pi n$$

- Josephson energy

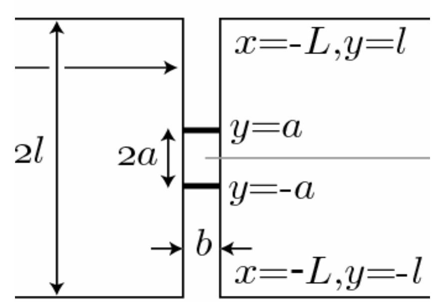
$$E \sim -(\cos\theta_1 + \cos\theta_2)$$

- Frustrated unless

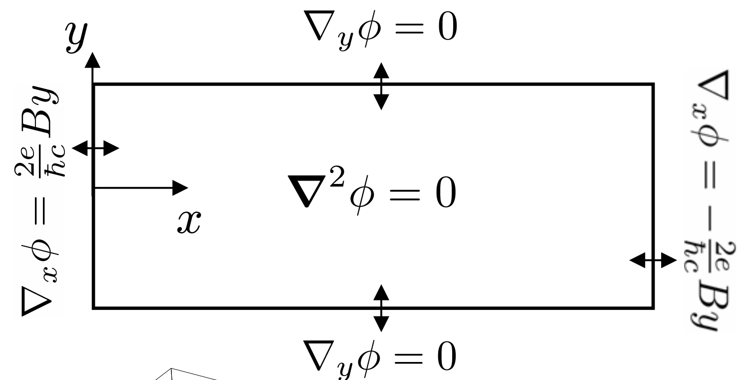
$$(8e/\hbar c)\, Bal \approx 2\pi n$$

- Period, to O(1)

$$B_{\text{fast}} \approx \frac{\pi\hbar c}{4eal} \approx \frac{\Phi_0}{4al}$$



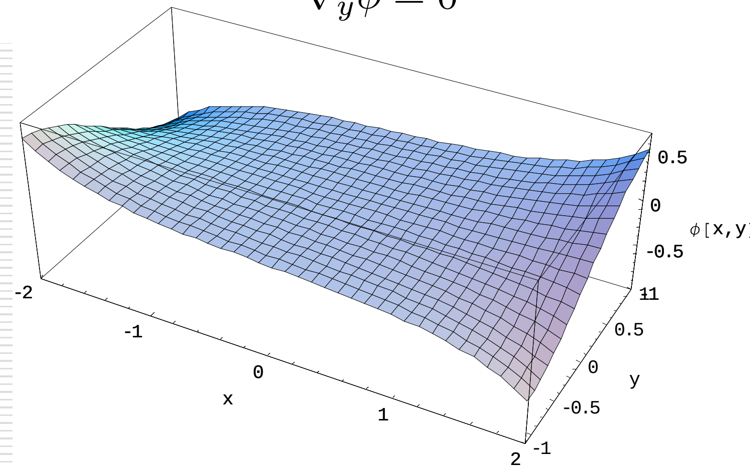
Precise periodicity



- Phase profile in lead $\Psi(\boldsymbol{r}) = \Psi_0 \, e^{i\phi(\boldsymbol{r})}$
- GL equation $(\boldsymbol{A}(\boldsymbol{r}) = By\,\boldsymbol{e}_x)$

$$\alpha\Psi + \beta|\Psi|^2\Psi + \frac{1}{2m}\left(\frac{\hbar}{i}\boldsymbol{\nabla} - \frac{2e}{c}\boldsymbol{A}\right)^2 \Psi = 0$$

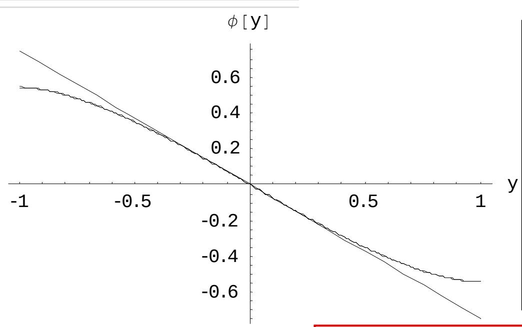
$$\Rightarrow \boldsymbol{\nabla}^2\phi = \frac{2e}{\hbar c}\boldsymbol{\nabla}\cdot\boldsymbol{A} = 0$$

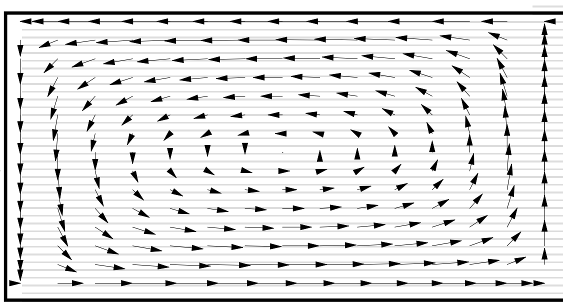


- Laplace's equation (with no current through boundary)

$$\boldsymbol{n}\cdot\boldsymbol{j}\Big|_\Sigma = 0, \quad \text{where} \quad \boldsymbol{j} = \left(\boldsymbol{\nabla}\phi - \frac{2e}{\hbar c}\boldsymbol{A}\right)$$

- Precise calculation…



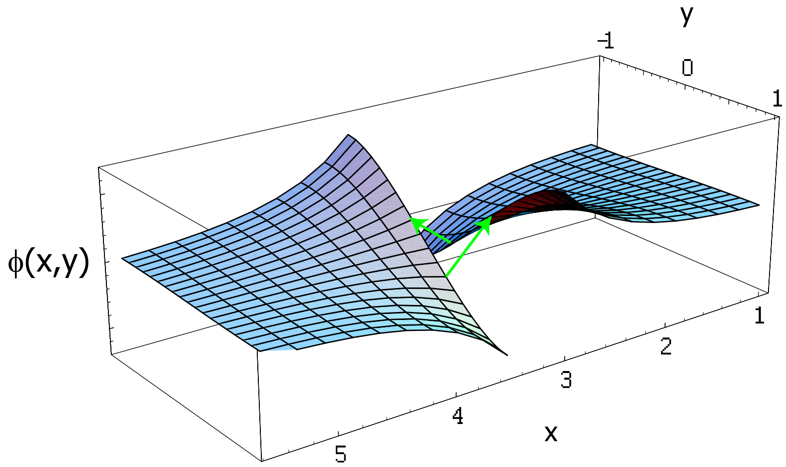


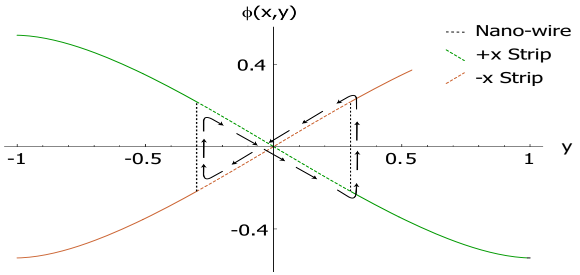
$$\delta(B) = \frac{32G}{\pi}\frac{Bla}{\Phi_0}$$

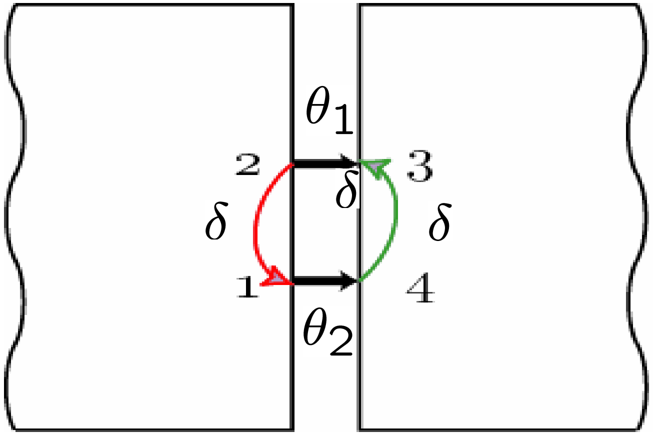
$$B_{\text{fast}} = \frac{\pi^2}{8G}\frac{\Phi_0}{4al}$$

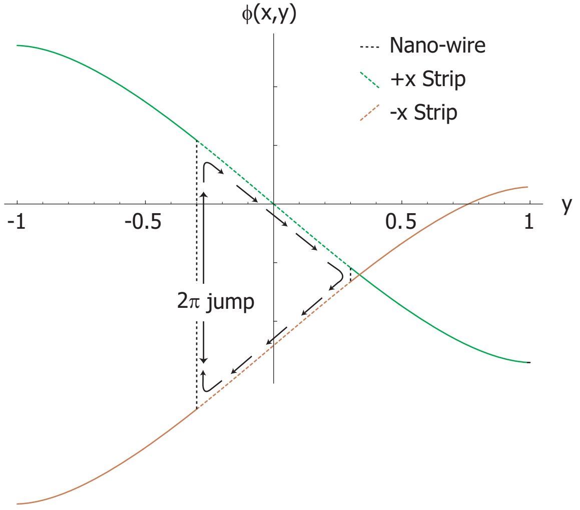
Catalan number $G = 0.916\ldots$

Low-field regime: Review of periodicity

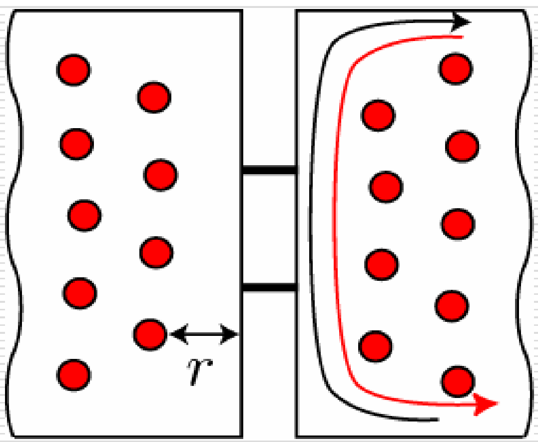








High-field regime: Role of vortices

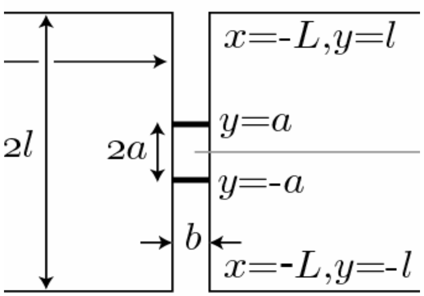


Vortex-free regime

- ignore flux through lead-bridge contour

$$al \gg ab$$

$$B_{\text{fast}} \approx \frac{\Phi_0}{4al}$$



Vorticial regime

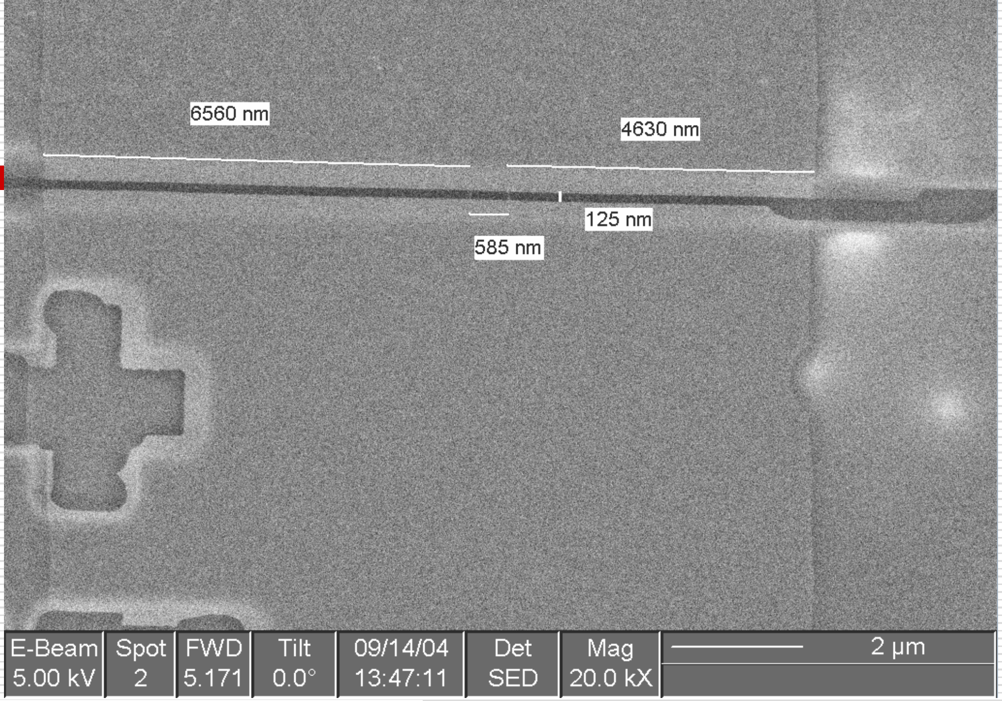
$$l \to r$$

$$2ar \lesssim 2ab$$

- usual (AB) period

$$B_{\text{sl}} \approx \frac{\Phi_0}{2ab}$$

Period? Sample 219-4



- all dimensions in nm

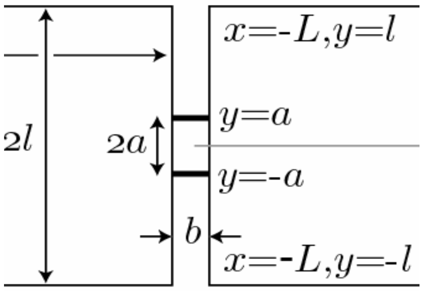
$$2l = (6,560 + 585 + 4,630)\ \text{nm}$$
$$2a = 585\ \text{nm}$$
$$b = 125\ \text{nm}$$

$$B_{\text{fast}} = c_o \frac{\pi^2}{8G}\frac{\Phi_0}{4la} = 412\ \mu\text{T} \qquad B_{\text{fast}}^{\text{exp}} = 450\ \mu\text{T}$$

$$B_{\text{slow}} \approx \frac{\Phi_0}{2ab} = 28\ \text{mT} \qquad B_{\text{slow}}^{\text{exp}} = 10\ \text{mT}$$

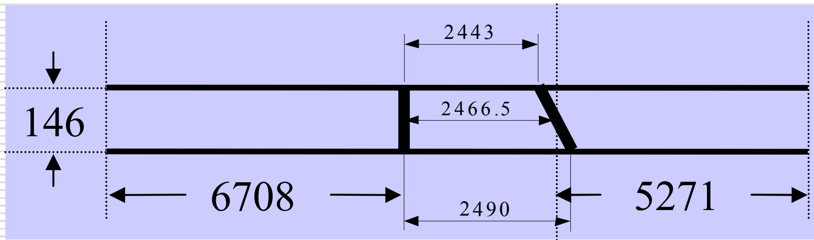
$$H_{\text{s}} \approx \frac{1}{\pi}\frac{\Phi_0}{2l\,\xi(T)} = 5.5\ \text{mT} \qquad H_{\text{s}}^{\text{exp}} = 2.5\ \text{mT}$$

- off-centre compensation $c_0 = 1.02$
- reasons for discrepancy?
  - “+” shaped hole in sample



Period? Sample 930-1/uncut

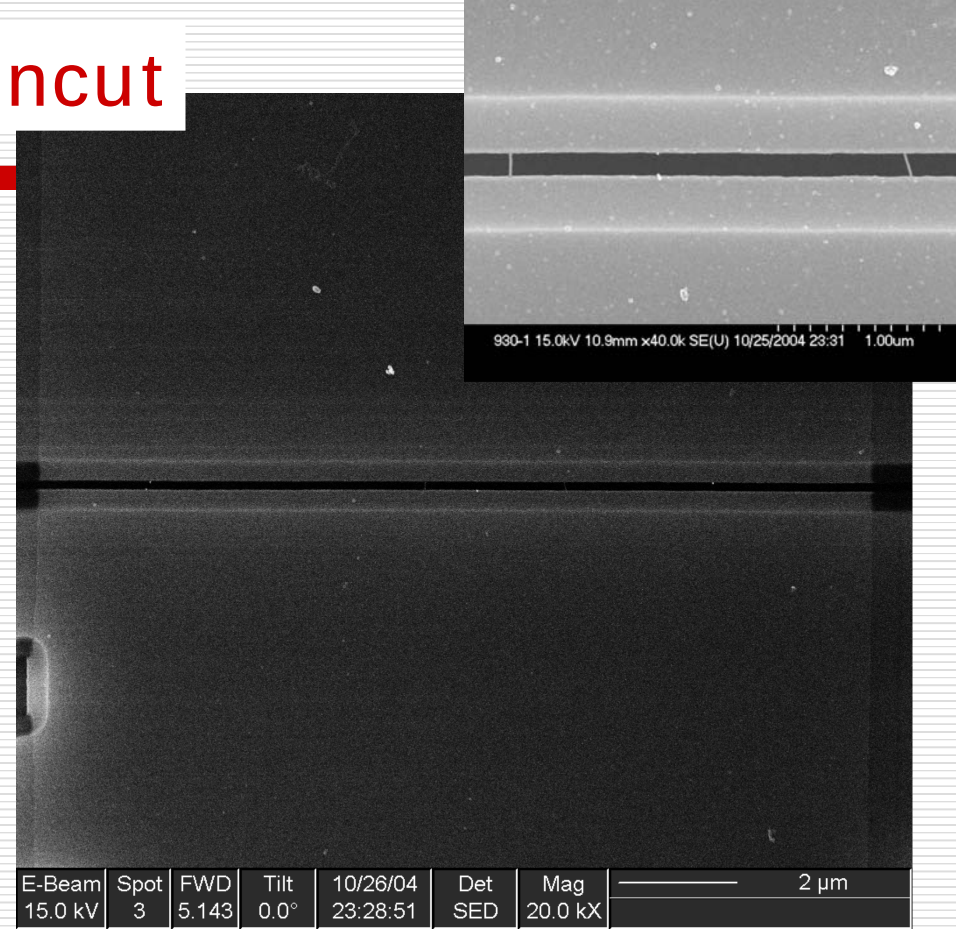
- all dimensions in nm



$$2l = (6,708 + 2,467 + 5,271)\ \text{nm} = 14,446\ \text{nm}$$

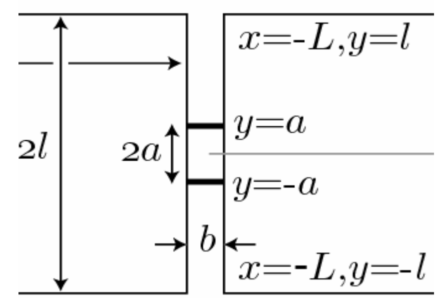
$$2a = 2,467\ \text{nm}$$

$$b = 146\ \text{nm}$$



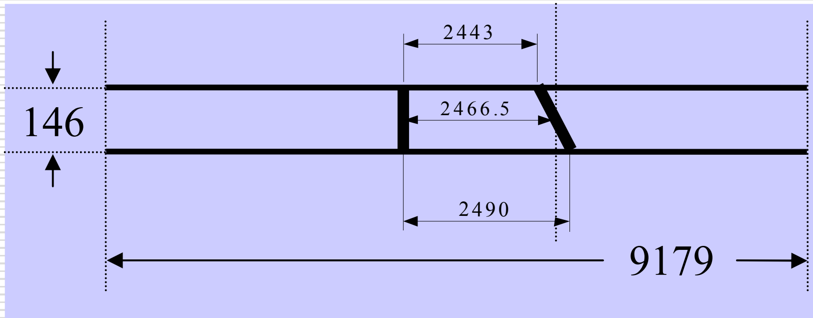
Magnetoresistance period

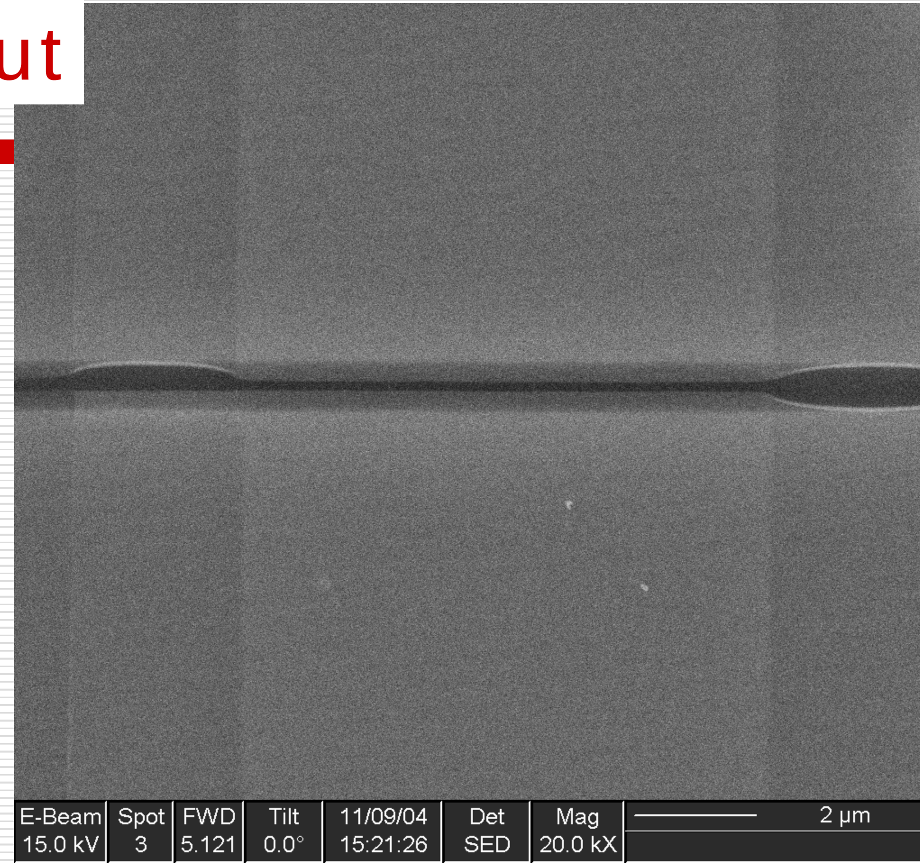
- experiment: 77.6 ± 0.1 µT
- ignoring bridge/lead contour: 78.18 µT
- with fundamental contour: 77.13 µT



Period? Sample 930-1/cut

- all dimensions in nm

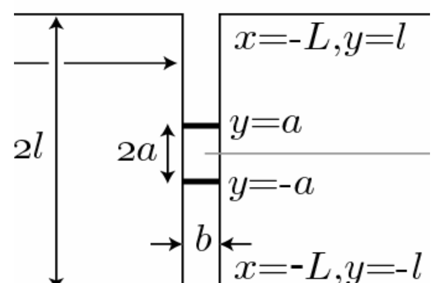




$$2l = 9,178 \text{ nm}$$
$$2a = 2,467 \text{ nm}$$
$$b = 146 \text{ nm}$$

Magnetoresistance period

➢ experiment:	128	μT
➢ ignoring fundamental contour:	123	μT
➢ with fundamental contour: (impact of cutting?)	120.5	μT

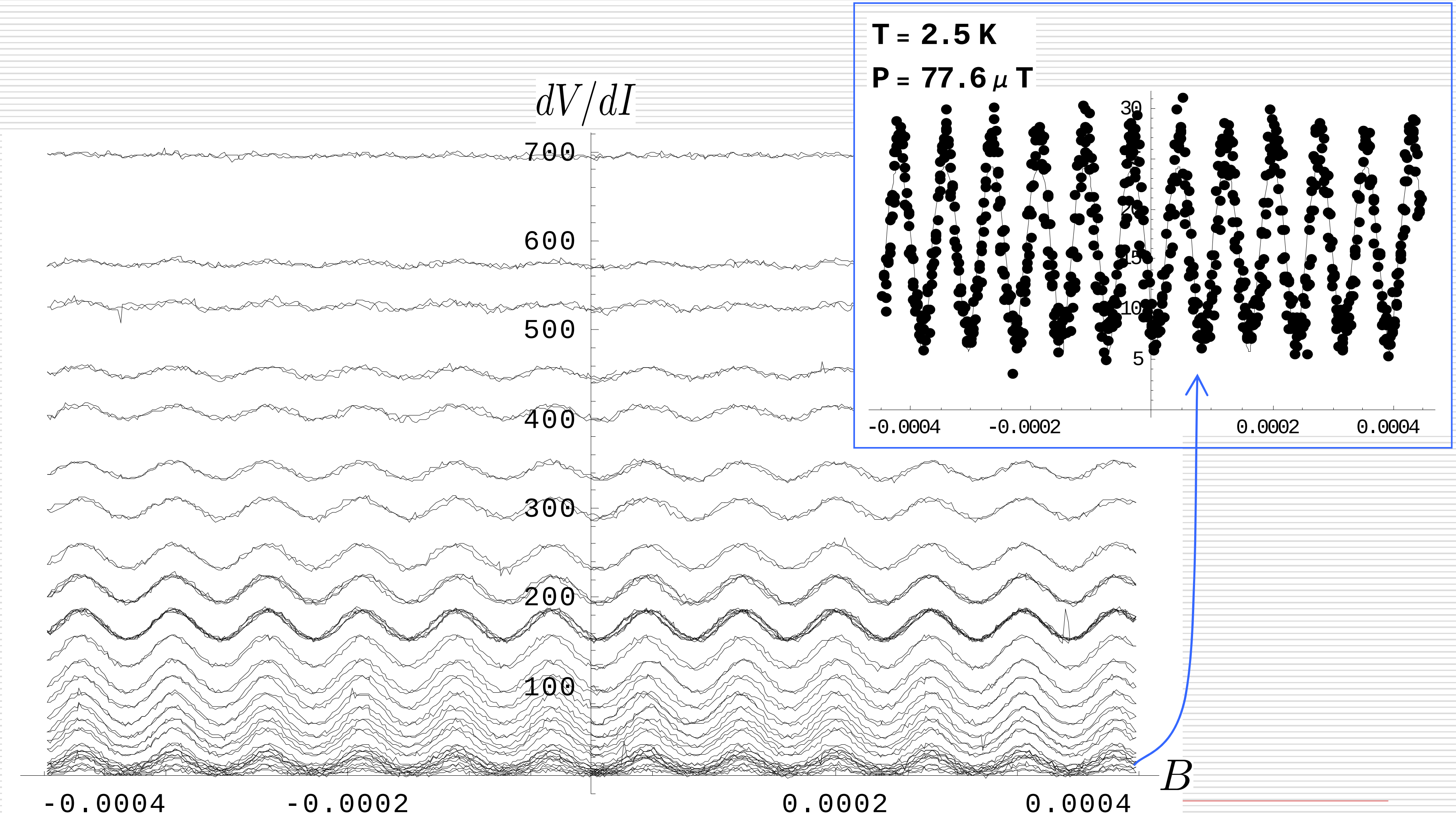


Period: Summary of samples

Sample	Wire len. (nm)	Wire sep. (nm)	Lead width (nm)	Th. per. (μT)	Ex. per. (μT)	% diff
205-4	121	265	11,270	929	947	1.9%
219-4	137	595	12,060	389	456	14.8%
930-1	141	2,450	14,480	78.4	77.5	-1.2%
930-1 (cut)	141	2,450	8,930	127	128	0.9%
205-2	134	4,050	14,520	47.4	48.9	3.0%

Sample 930-1/uncut: Resistance…

vs. magnetic field & temperature



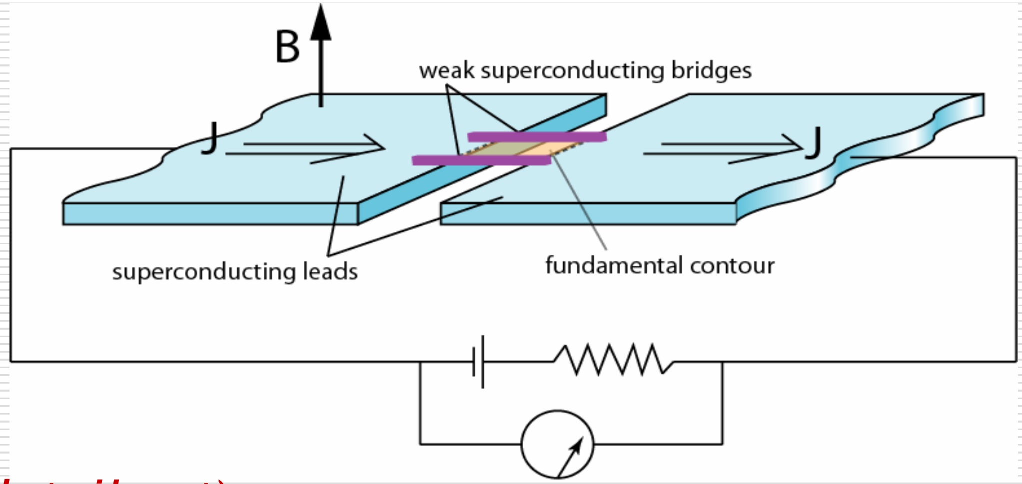
So far…

- What **controls** resistance?
  magnetic field
  $\Longrightarrow$state of leads
  $\Longrightarrow$state of bridges
  negligible feedback on leads?
- Yields…
  - regimes cross-over field
  - oscillation periods (low- & high-field regimes)
- But what about…
  magnetic field & temperature dependence…
  - of resistance magnitude?
  - of oscillation amplitude?
- Need to focus on **mechanism** for resistance itself

Mechanism for resistance

Elements of a model

- 2-wire device (*arrays*?), fixed total current
- magnetic field controls leads
- leads influence bridges (*feedback*?)
- intrinsic resistance of bridges via dissipative thermal fluctuations of s/c order parameter
  - *Josephson junctions*
  - *LA-MH*



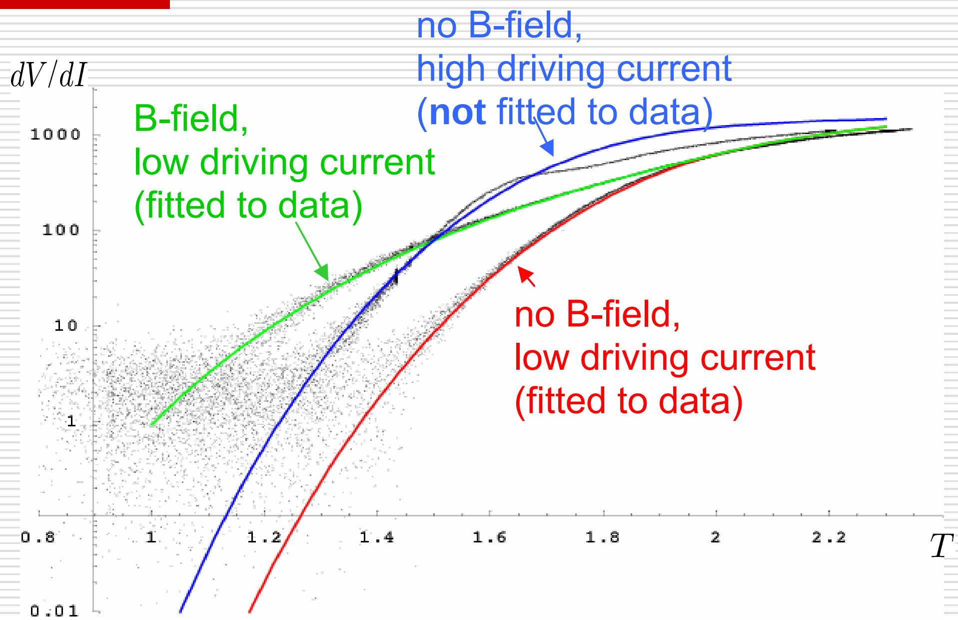
Yield?

- magnitude of resistance
- temperature dependence
- magnetic-field dependence
- current dependence (*not in detail yet*)

Sample 219-4/930-1: Resistance...

sample 219-4
short wire-length regime:

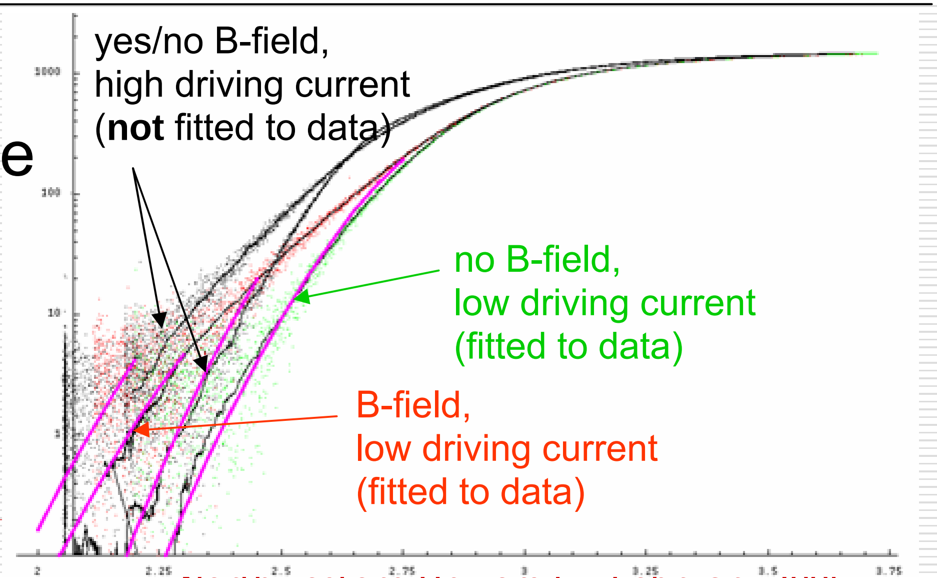
j_{c1} = 639 nA
j_{c2} = 330 nA
T_{c1} = 2.98
T_{c2} = 2.00



sample 930-1
intermediate wire-length regime

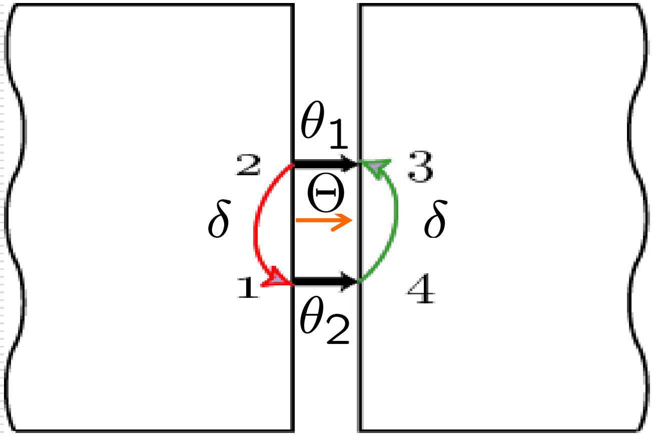
RN1 = 2882.9 Ω	Tc1 = 3.147	ξ01 = 17.29 nm
RN2 = 2941.7 Ω	Tc2 = 3.716	ξ02 = 8.700 nm

$Length1 = 146 * 10^{-9}$;
$Length2 = 150 * 10^{-9}$;



Thermodynamic variables? (JJ or LA-MH)

- Voltage controlled?
  - lead phase difference Θ as independent variable
  - Helmholtz free energy $F(\Theta)$
- Current controlled?
  - total current $I = I_1 + I_2$ as independent variable
  - Gibbs free energy $G(I) = F(\Theta) - (\hbar/2e)I\Theta$
- Rigid leads $F(\Theta) = F_1(\theta_1) + F_2(\theta_2)$
- Phase constraint $\theta_1 - \theta_2 = 2\pi n + 2\delta$
- Total current constraint $I = I_1(\theta_1) + I_2(\theta_2)$



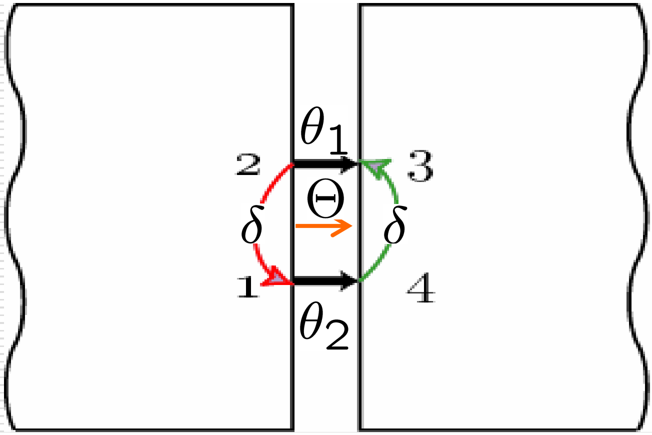
cf. McCumber, Phys. Rev. **172**, 427 (1968)

Josephson junction model I

- Helmholtz free energy per junction

$$F_1(\theta_1) = -\frac{\hbar}{2e} I_{c1} \cos\theta_1 \qquad \theta_1 = \Theta + \delta$$

$$F_1(\theta_2) = -\frac{\hbar}{2e} I_{c2} \cos\theta_2 \qquad \theta_2 = \Theta - \delta$$



- Gibbs free energy

$$G = -\frac{\hbar}{2e}\left(I_{c1}\cos(\Theta+\delta) + I_{c2}\cos(\Theta-\delta) + I\,\Theta\right)$$

- Gives single effective junction

$$G = -\frac{\hbar}{2e}\left(\sqrt{(I_{c1}+I_{c2})^2\cos^2\delta + (I_{c1}-I_{c2})^2\sin^2\delta}\cdot\cos\Theta + I\,\Theta\right)$$

$$\Theta \to \Theta + \tan^{-1}\left[\frac{I_{c1}-I_{c2}}{I_{c2}+I_{c1}}\tan\delta\right]$$

Josephson junction model II

$$G = -\frac{\hbar}{2e}\left(\sqrt{(I_{c1}+I_{c2})^2\cos^2\delta + (I_{c1}-I_{c2})^2\sin^2\delta}\cdot\cos\Theta + I\,\Theta\right)$$

$$\Theta \to \Theta + \tan^{-1}\left[\frac{I_{c1}-I_{c2}}{I_{c2}+I_{c1}}\tan\delta\right]$$

- Apply IZ-AH single-junction theory to **effective** junction

Ivanchenko & Zil'berman, JETP Lett. **8**, 113 (1968)
Ivanchenko & Zil'berman, JETP **28**, 1272 (1969)
Ambegaokar & Halperin, PRL **22**, 1364 (1969)

- Barrier crossing approximation gives…

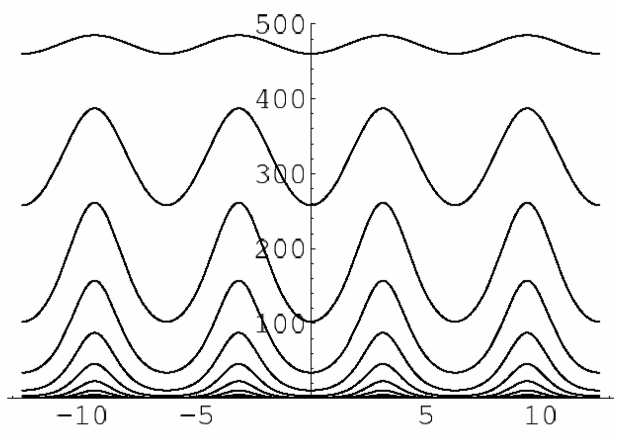
$$R = 2R_n\sqrt{(x^{-2}-1)}\,e^{-\gamma(\sqrt{1-x^2}+x\arcsin x)}\sinh\frac{\pi\gamma x}{2}$$

$$I_{\rm eff} = \sqrt{(I_{c1}+I_{c2})^2\cos^2\delta + (I_{c1}-I_{c2})^2\sin^2\delta}$$

$$x \equiv I/I_{\rm eff} \qquad \gamma \equiv \hbar I_{\rm eff}/eT \qquad \delta = (32G/\pi)(Bla/\Phi_0)$$

- Can solve Fokker-Planck equation exactly (*cumbersome but used for numerics*)

Dim'less resistance for various temps & fields



Josephson junction model III

- Temperature dependence of effective critical current?
  - *for junctions model*: from T_c's of leads
  - *for wires model*: T_c's of wires

- Critical current for a wire (*Tinkham*)

$$I_c = \frac{8\, e\, \sigma H_c^2(T)\xi(T)}{3\sqrt{3}\,\hbar\mu_0} \sim \left(1 - \frac{T}{T_c}\right)^{3/2}$$

- Critical currents for junctions

$$I_{c1}(T) = I_{c1}\left(1 - \frac{T}{T_{c1}}\right)^{3/2} \qquad I_{c2}(T) = I_{c2}\left(1 - \frac{T}{T_{c2}}\right)^{3/2}$$

- Justification: maximum number of twist per wire is small

$$n_{\max} = b/2\pi\sqrt{3}\,\xi(T) \lesssim 1 \qquad b = 125\,\text{nm}, \;\; \xi_0 = 12\,\text{nm}$$

- Parameters in model

$I_{c1},\, T_{c1},\, I_{c2},\, T_{c2}$ fitted

R extracted from experiment

Sample 219-4/930-1: Resistance...

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short wire-length regime:

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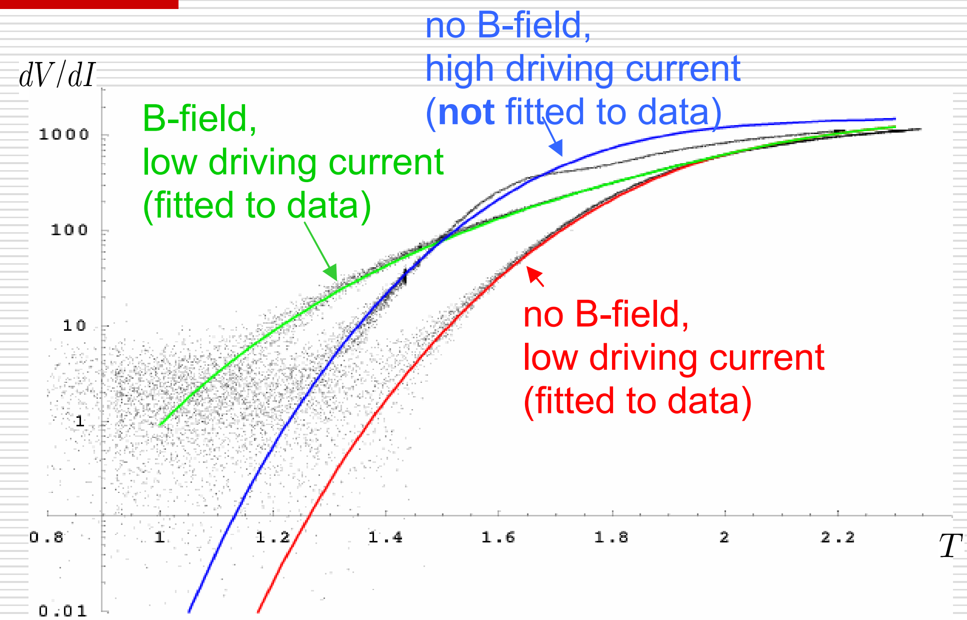

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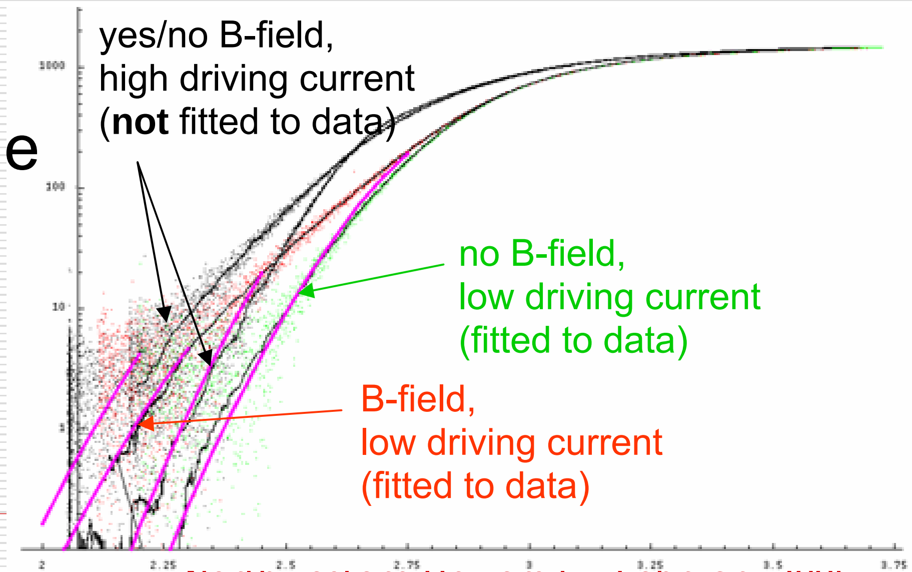
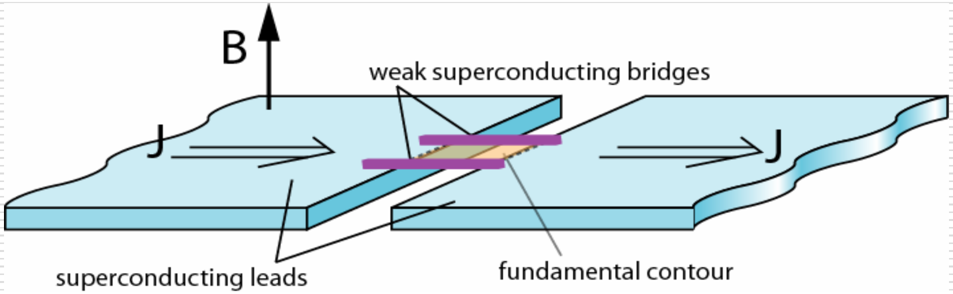


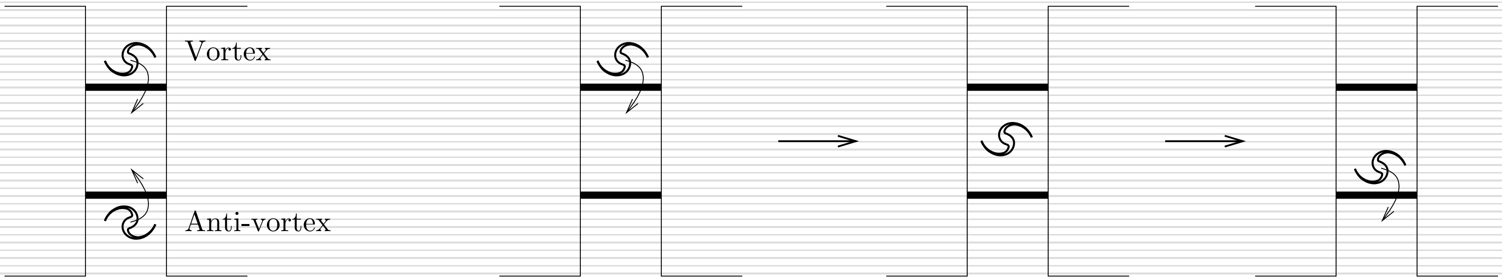
Improved model: Wires instead of junctions

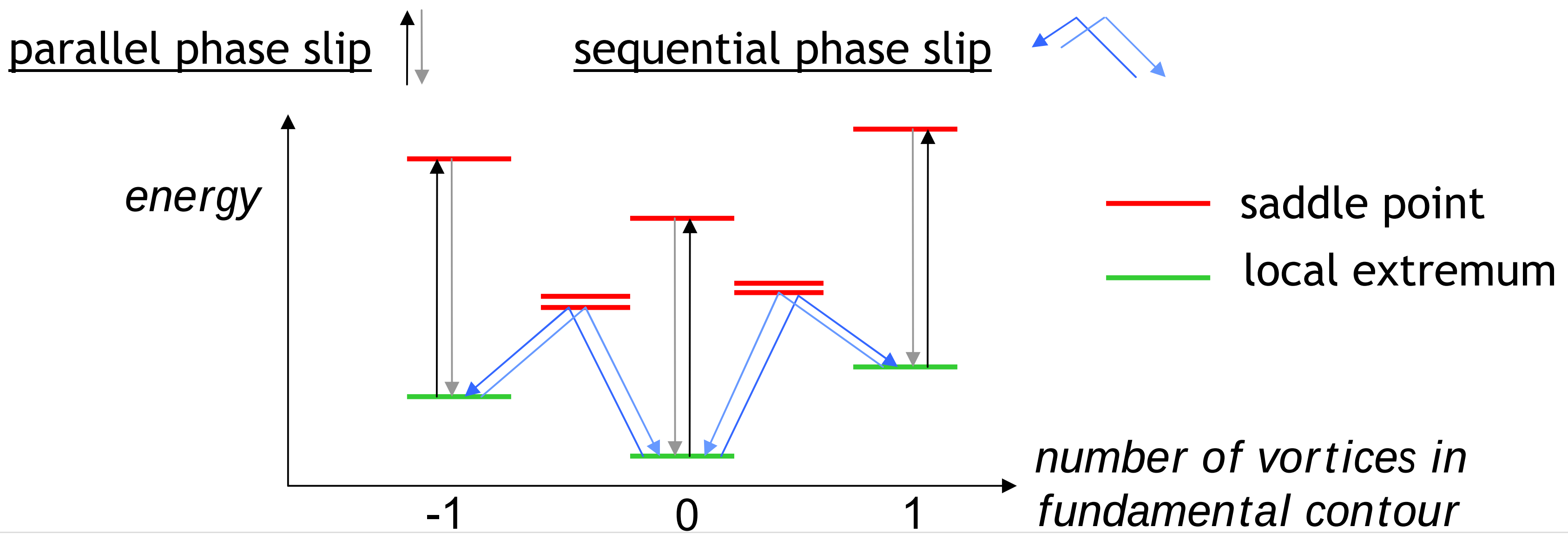


- Theoretical motivation
  - wires are short, but…
  - can support some twist
  - so LA-MH theory a better framework
- Experimental motivation
  - fitting parameters uncomfortable
- Essential change
  - replace Josephson junctions by narrow wires
- Technical complications
  - simple phases $\Longrightarrow$ extended freedoms
  - variety of resistive processes: which dominate?
  - some analytical tractability lost
  - but broader range of validity

Two-wire system: Extension of LA-MH

- Effective inter-wire coupling in LA-MH barrier-crossing model
- *Coupled instantons*: analytically tractable for long wires & low currents
- What processes to consider? Analyze dynamics via Markov chain





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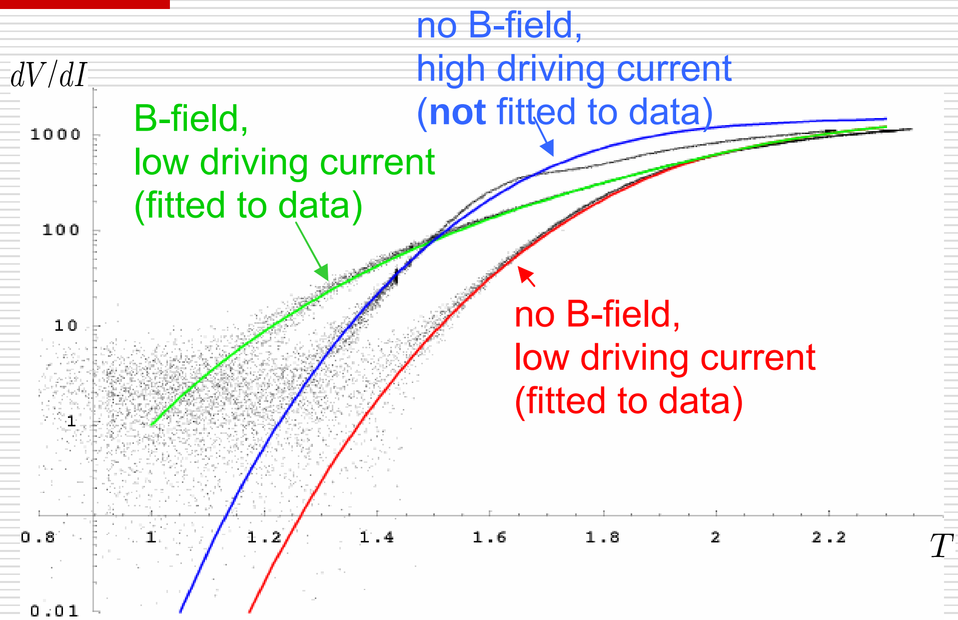

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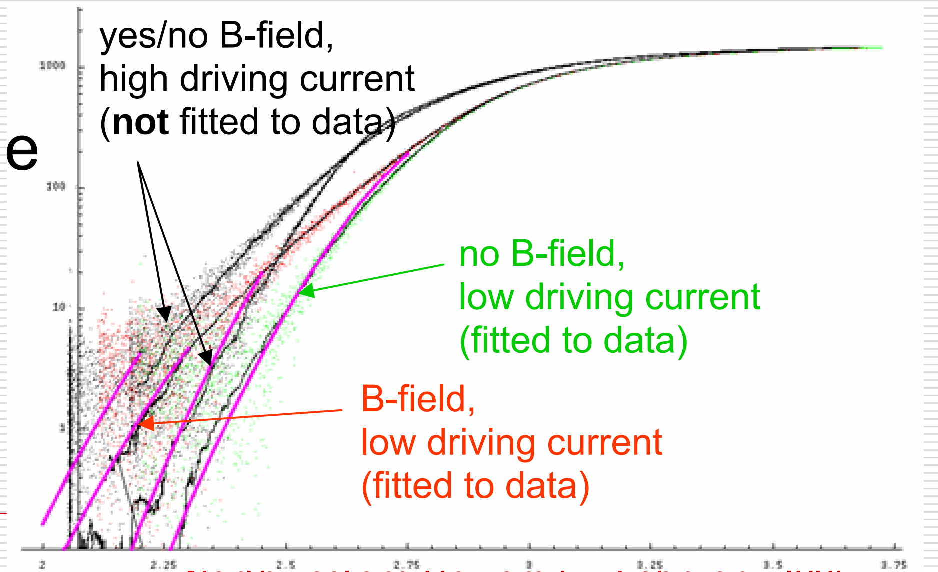
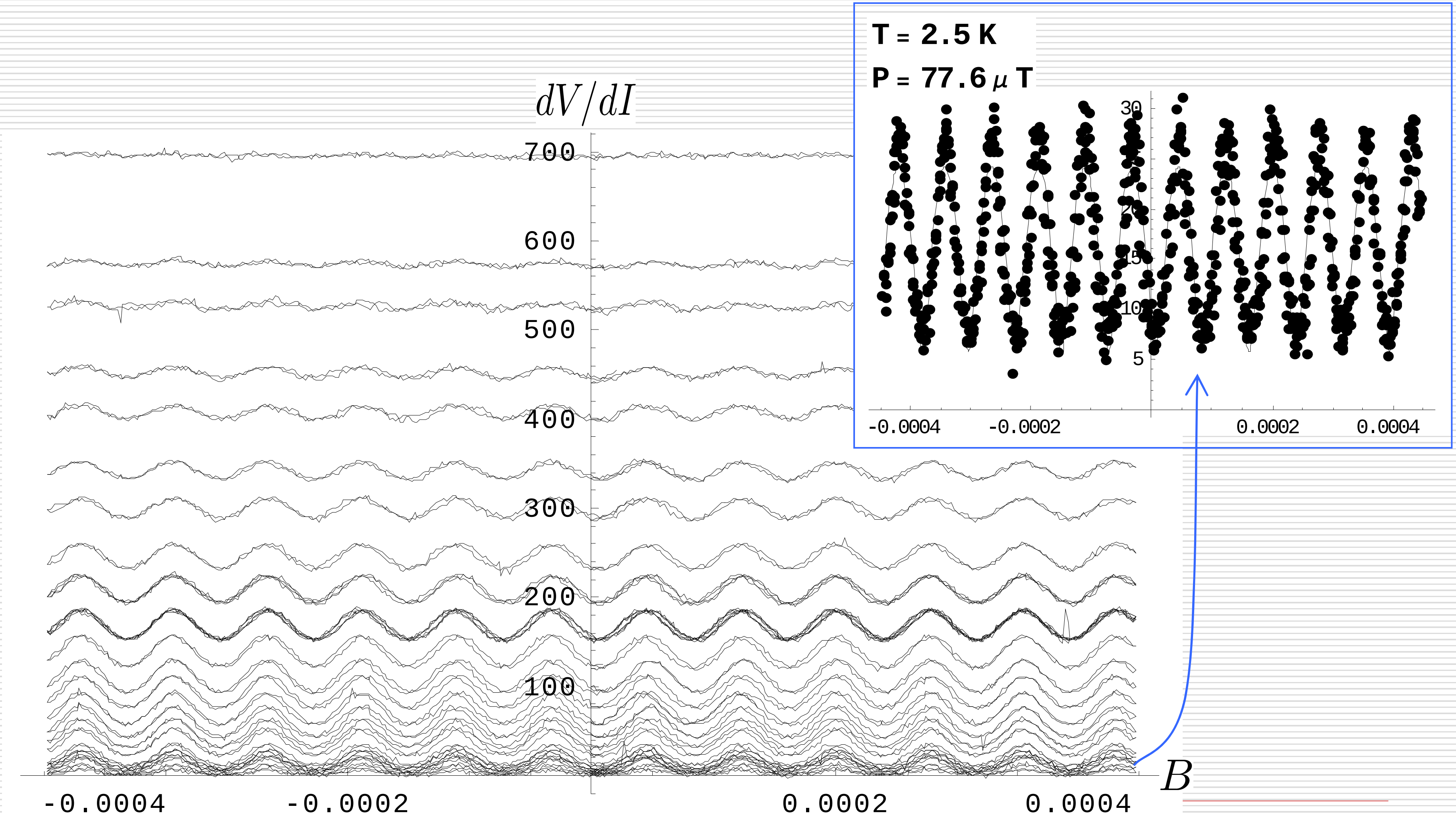


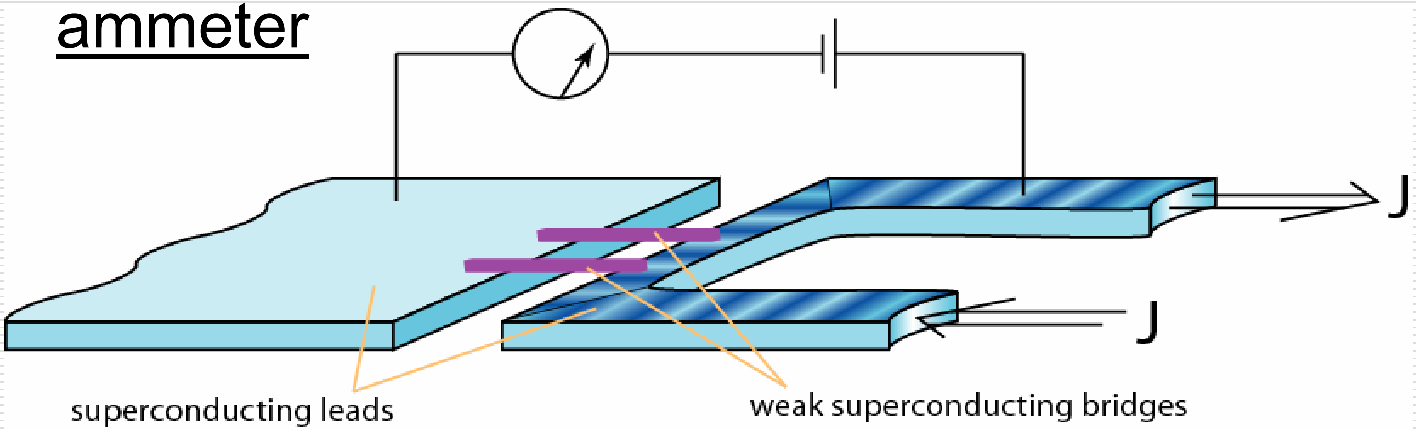
Sample 930-1/uncut: Resistance…

vs. magnetic field & temperature

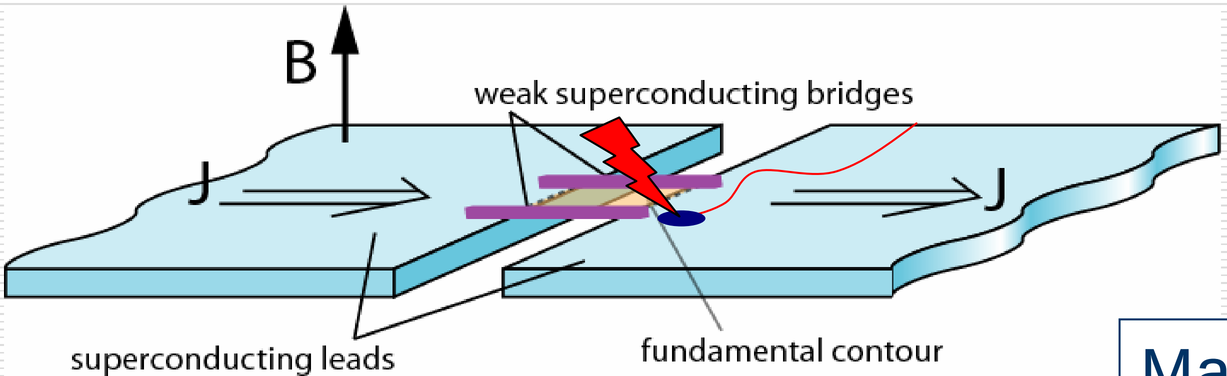


Gradiometer-type experiment? Examples

ammeter

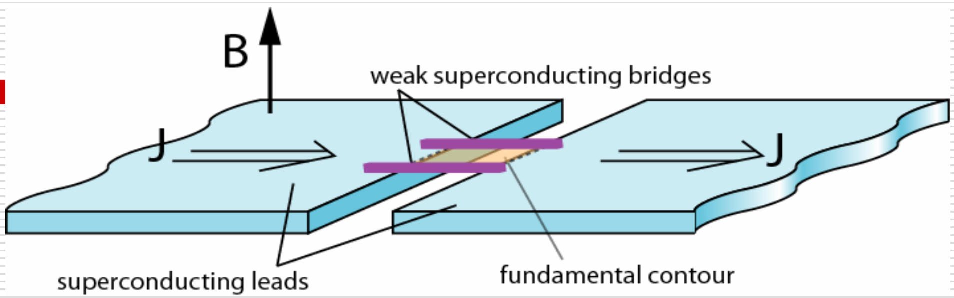


vortex tracking



Mapping out the s/c order parameter

Concluding remarks



- Hopkins/Bezryadin experiments
  - SQUID via DNA templating
  - nanoscale structure
  - mesoscale properties
- Origins of device resistance
- Field regimes, period of resistance (with magnetic field)
  - back-of-the-envelope picture
  - more precision
- Temperature- and current-dependence
  - simple model: Josephson junction phase fluctuations
  - refinements: LA-MH order parameter fluctuations
- Comparisons with experiments
- Mapping out superconductivity?