Dependence of global superconductivity on inter-island coupling in arrays of long SNS junctions

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Abstract
We present measurements of the superconducting transition temperature, T_c, for arrays of mesoscopic Nb islands patterned on Au films, for large island spacings d. We show that $T_c \sim 1/d^2$, and explain this dependence in terms of the quasiclassical prediction that the Thouless energy, rather than the superconducting gap, governs the inter-island coupling at large spacings. We also find that the temperature dependence of the critical current, $I_c(T)$, in our arrays is similar to that of single SNS junctions. However, our results deviate from the quasiclassical theory in that T_c is sensitive to island height, because the islands are mesoscopic.

(Some figures may appear in colour only in the online journal)

Arrays of superconducting islands placed on thin normal-metal films offer tunable realizations of two-dimensional superconductivity [1–3]. In such superconductor–normal-metal–superconductor (SNS) arrays, the superconducting islands are coupled to one another through the proximity effect, which is known to give rise to global phase coherence through a Berezinskii–Kosterlitz–Thouless (BKT) transition [1, 2, 4–6]. Understanding the dependence of the superconducting critical temperature, T_c, on island spacing can shed light on the energy scales, and thereby the physical processes, governing the inter-island coupling. The standard theoretical account of the spacing dependence of T_c, due to Lobb, Abraham, and Tinkham (LAT) [2], predicts that T_c should depend on island spacing d according to the relation $T_c \approx \Delta \exp(-d/\xi_N(T_c))$, where Δ is the superconducting gap on the islands, $\xi_N(T) = \sqrt{\hbar D/k_B T}$ is the normal-metal coherence length, and D is the normal-metal diffusion constant. However, there are two important limitations to the

LAT analysis, which might lead to a sharper falloff of T_c with d. First, the LAT analysis is based on the Ginzburg–Landau theory, which is valid only when Δ is smaller than any other relevant energy scale. However, for sufficiently widely spaced islands, Δ is much larger than the Thouless energy $E_{Th} = \hbar D/d^2$ (which is inversely proportional to the time for the diffusion of electrons between the islands). In this regime, the quasiclassical theory [7, 8] predicts that the proximity coupling between islands depends only on E_{Th} and not on Δ; i.e., that global superconductivity depends only on the properties of the normal-metal film and not on those of the superconducting islands [7–10]. As we argue below, the quasiclassical theory then implies that $T_c \sim 1/d^2$. Second, the LAT analysis is mean-field in character, and therefore ignores fluctuations that can lead to anomalous suppression of T_c, and possibly even a zero temperature metallic state for very widely spaced islands [11–13].

To test these predictions, we have systematically studied the dependence of the superconducting transition temperature on island spacing for arrays of widely spaced islands. In a recent paper [14], we showed that the spacing dependence
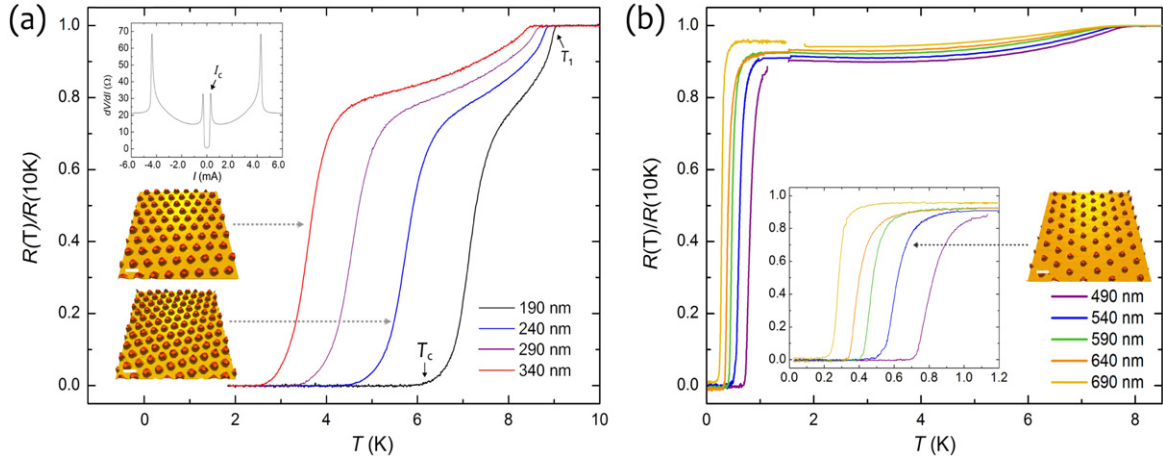
Figure 1. (a) Resistance R (normalized to values at 10 K) versus temperature T for four arrays, each having different island spacings d. The island heights are 145 nm. For the array having 190 nm-spaced islands, the transition temperature of the islands is marked as T_1 and superconducting transition across the array as is marked as T_c. Lower inset: AFM images of Nb islands (red) on Au films (yellow) for each indicated array (scale bar is 500 nm). Upper inset: typical differential resistance dV/dI versus bias current characteristics for $T < T_c$; here, $d = 190$ nm and $T = 2.4$ K. The inner peaks, denoted I_c, correspond to the critical current of the proximity-coupled film; the outer peaks correspond to the critical current of the islands. (b) Similar normalized R versus T curves for more widely spaced islands. The island heights are 125 nm. Data were collected in two different cryostats, resulting in a discontinuity around 1.4 K. Center inset: a zoomed-in plot of the low-temperature $R(T)$ behavior presented in the main panel.

of T_c deviated from the LAT prediction for islands having $d < 340$ nm; however the $1/d^2$ dependence was not as clear for these relatively closely spaced islands. In this paper, we measure T_c for islands spaced up to 690 nm (i.e., in the regime where $E_{Th} \ll \Delta$), and find that it clearly depends on spacing as $1/d^2$, which is consistent with the Δ-independent behavior predicted by the quasiclassical theory. The dependence of T_c on island spacing in SNS arrays directly exposes the role of the Thouless energy: because the quasiclassical theory implies that the only relevant energy scales are E_{Th} and $k_B T$, it follows from dimensional analysis that T_c must be proportional to E_{Th}, and thus that $T_c \sim 1/d^2$. In addition, we present measurements of the temperature dependence of the critical current $I_c(T)$, and find that the behavior of $I_c(T)$ in arrays is similar to that of single SNS junctions and SQUIDs [7, 9, 15]; such measurements of I_c are also consistent with quasiclassical predictions. Finally, we show that our results deviate from the quasiclassical predictions: T_c depends nontrivially on the height of the islands, which reflects the mesoscopic character of the islands, and distinguishes our experiments from those of [1].

The devices studied in this paper consist of triangular arrays of Nb islands (see figure 1) fabricated on 10-nm thick Au films, which are patterned for four-point measurements, on Si/SiO$_2$ substrates [14]. Each substrate contains up to six film/array devices, which are identical except for their systematically varied island spacings. The diameter of each Nb island is 260 nm; the island height (i.e., Nb thickness) is identical for all devices on a single substrate, but ranges from 87 to 145 nm for the sets of devices presented in this paper. The islands are composed of columnar grains $\sim$30 nm in diameter [14], and have a dirty-limit coherence length $\xi_0^{Nb} \approx 27$ nm, for an approximate mean free path $\ell \approx 8$ nm. The edge-to-edge island spacings d

range from 140 nm to 690 μm and the number of islands per array varies from 11 400 to 155 800. The Au film has an estimated diffusion constant $D \approx 95$ cm^2 s^{-1} for a mean free path of $\ell \approx 13$ nm and a temperature-dependent coherence length $\xi_N(T) \approx 207$ nm/$\sqrt{T}$, where T is in units of K. Resistance measurements (through the Au film) were performed using standard low frequency, ac lock-in techniques in either a pumped He-4 cryostat, a He-3 cryostat, or a He-3/He-4 dilution refrigerator. Figure 1 shows the temperature-dependent normalized resistance for arrays having islands spaced 190–690 nm apart. The islands become superconducting at a transition temperature T_1, and superconductivity across the array appears below a critical temperature $T_c < T_1$ [14]. The island height in all samples is verified using atomic force microscopy. For a given Nb island spacing and height, and Au layer thickness, the transition temperatures are found to be reproducible, even for devices prepared on different substrates.

The upper inset to figure 1 shows the current-biased differential resistance dV/dI for an array having $d = 240$ nm at $T = 2.4$ K (i.e., below T_c); the shape of the curve is typical of results observed in all of our arrays. Differential resistance was determined by differentiating the IV characteristics. To minimize the effects of Joule heating at high currents (e.g., for $I > I_c$), these IV characteristics were measured using current pulses, with a current-on time of 3.5 ms and -off time of 3 ms. Peaks are evident at positive and negative currents; the lower-current (inner) peaks mark the critical current I_c across the proximity-coupled Au film (i.e., where the IV curves become approximately Ohmic) [16]. The higher-current (outer) peaks correspond to the critical current of the islands. In this paper, we focus on the inner peaks, or the I_c corresponding to the superconducting transition of the film. Figure 2(a) shows these inner peaks for devices
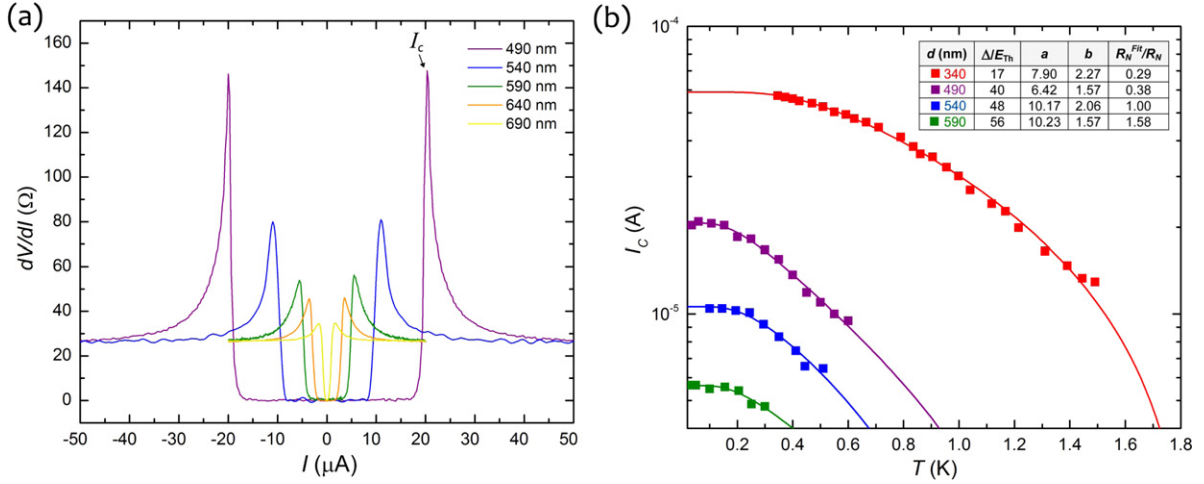
Figure 2. (a) Differential resistance versus bias current as a function of island spacing, at $T \approx 30$ mK. The critical current I_c is extracted from the peak position. (b) I_c versus T, for four island spacings. Solid lines are fits to the quasiclassical theory, as discussed in the text; parameters relevant to the fits are given in the table.

having various island spacings. As can be seen in the figure, the peaks move toward lower bias—i.e., I_c decreases—for larger island spacings. In figure 2(b), we show I_c as a function of temperature T for four different arrays, where I_c was extracted from the peaks in dV/dI. The solid curves are fits to the temperature-dependent, quasiclassical equations for a single, diffusive SNS junction, namely $I_c(T) = a\frac{E_{Th}}{eR_N}(1 - be^{-aE_{Th}/3.2k_BT})$, where R_N is the normal resistance, and dimensionless parameters a and b depend on Δ/E_{TH} [7]. We treat R_N as a fitting parameter, R_N^{Fit} noted in the table in figure 2, which deviates somewhat from the actual R_N; this deviation has been previously observed in numerous studies of single, diffusive SNS junctions [7, 9, 17–19]. As can be seen in figure 2(b), the quasiclassical predictions fit the data well, and the fit values (noted in the table in figure 2) are close to those predicted for very widely spaced single junctions ($\Delta \gg E_{Th}$), namely $a = 10.82$ and $b = 1.30$, which suggests that the SNS arrays display behavior similar to that of single junctions.

We now discuss the dependence of T_c on island spacing. The value of T_c was extracted using two different methods. The first method involved extracting T_c from the *IV* isotherms (see figure 3(a)); these curves exhibit behavior consistent with a BKT transition at a critical temperature T_{BKT}, which is a hallmark of 2D superconducting systems. For this transition, it is predicted that above T_{BKT}, the system is Ohmic; for $T < T_{BKT}$, the *IV* characteristics are nonlinear such that $V \propto I^{\alpha(T)}$. We observe the 'Nelson–Kosterlitz' jump in $\alpha(T)$ from 1 to 3 and, following standard practice [6, 20–26], mark $T_c = T_{BKT}$ as the temperature at which $\alpha(T) = 3$. In figure 3(c), we plot the spacing dependence of T_c, which is well captured by the form $T_c \sim 1/d^2$, especially for the arrays having the most widely spaced islands. Because finite size effects [27–29] and weak, stray magnetic fields [30] can affect the slope of the low-current regions of the *IV* isotherms, we also extract a transition temperature directly from the temperature-dependent resistances curves, utilizing the standard practice of setting a resistive criterion below

the start of the transition [31–35]. The results are shown in figure 3(d), where the temperature at which the resistance falls to 5% of its normal state value is plotted as T_c. The two methods of extracting T_c yield very similar results, showing that the trend $T_c \sim 1/d^2$ is independent of our extraction technique. Further, the suppression of T_c with increasing island spacing is more rapid than that predicted by the LAT theory, as is evident in figure 3(c).

The observed trend of $T_c \sim 1/d^2$ implies that $k_BT_c/E_{Th}(d) =$ constant; moreover, the data show that the constant is of order unity. This linear relationship between T_c and E_{Th} follows from the quasiclassical prediction [36] that E_{Th} is the unique energy scale governing the properties of a long SNS junction. One can understand this prediction in the following heuristic terms [36, 37]. The proximity effect is a consequence of Andreev reflection [36], whereby normal-metal electrons with energy $\varepsilon < \Delta$ above the Fermi energy incident at the NS interface are retroreflected as holes; at the interface, the phase between the electron and retroreflected hole is set by the superconductor. As long as the electron and hole stay phase coherent with one another while propagating through the normal metal, they carry information about the superconducting phase and thus mediate the proximity effect. It can be shown [38] that, at zero temperature in a diffusive metal, an electron with energy ε above the Fermi surface dephases with its retroreflected hole after traveling a distance of order $L_0 = \sqrt{\hbar D/\varepsilon}$, where D is the diffusion constant in the normal metal; hence, only electrons with energy $\varepsilon \lesssim \hbar D/d^2 = E_{Th}$ carry superconducting phase information and can be involved in the proximity effect. (This analysis is valid in the long-junction limit, where $E_{Th} < \Delta$.) Thus, the Thouless energy is the unique energy scale determining the zero temperature properties of SNS arrays in the long-junction limit. A direct consequence of this is that any other array observable with units of energy must be proportional to the Thouless energy (so that, e.g., $I_cR_N \sim E_{Th}$ [7]). Using this fact, the spacing dependence of all observables can be established by simple dimensional analysis. At finite temperatures, thermal dephasing adds
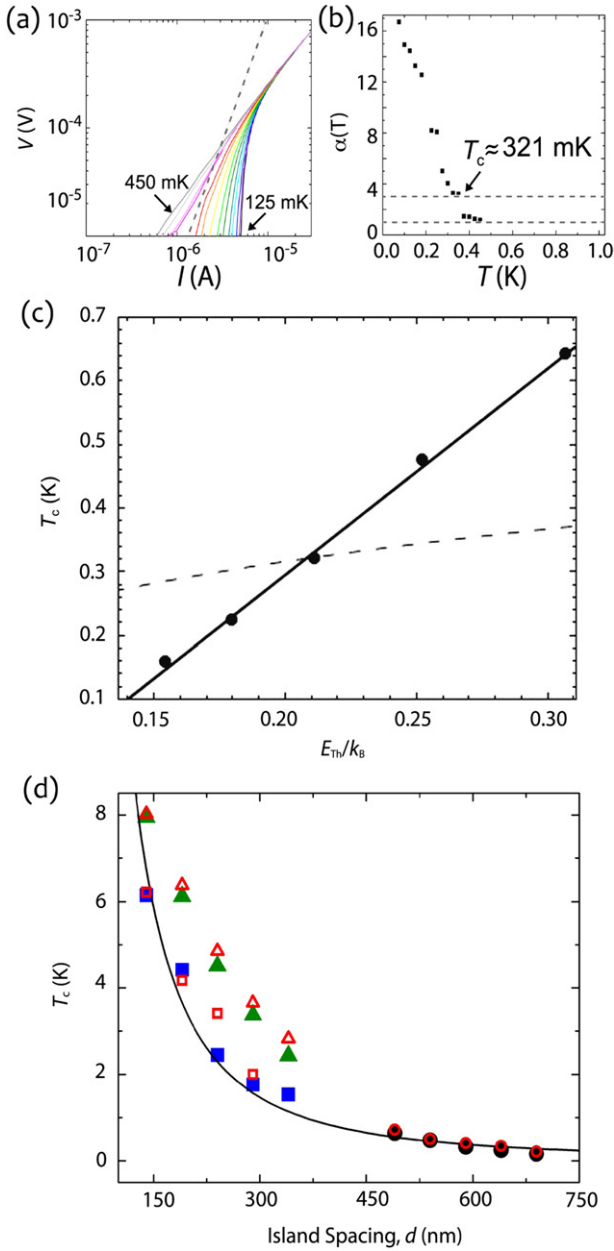
Figure 3. (a) Logarithmic current–voltage (*IV*) characteristics for 590 nm-spaced islands, at temperatures near T_c, ranging from 125 to 450 mK at intervals of 25 mK; dashed line separates Ohmic behavior above T_c from non-Ohmic ($V \sim I^{\alpha(T)}$) behavior below T_c. (b) The exponent α as a function of temperature for the same array. (c) T_c versus the Thouless energy $E_{Th} \sim 1/d^2$ for widely spaced islands ($d > 450$ nm). Solid black line shows linear fit. Dashed black line shows the fit to the LAT theory. (d) T_c versus d over a wider range of spacing. Squares, triangles, and circles correspond to island heights of 87 nm, 145 nm, and 125 nm, respectively, for which T_c is extracted from the *IV* isotherms as the temperature at which $\alpha = 3$ [20]. Open, red symbols represent T_c as extracted from another method—by taking the temperature at which the resistance falls to 5% the normal resistance, $R(10$ K). (Note that $R(T)$ for the 240 nm-spaced islands having 87 nm tall Nb is anomalous in shape [14], leading to marked difference in the extracted values for T_c from both techniques.) Solid black line shows the fit to $1/d^2$ for the black circles; the fit is also good for the blue squares. Note that T_c depends on island height.

an additional energy scale $k_B T$. Therefore, by dimensional analysis, all finite temperature properties are determined by the ratio E_{Th} to $k_B T$ (which can be equivalently rearranged as the ratio of ξ_N to d). In particular, this holds true for the transition temperature, so that $T_c \sim E_{Th}/k_B \sim 1/d^2$, which is indeed the trend we observe experimentally.

The quasiclassical theory not only predicts that $T_c \sim 1/d^2$, but also implies that the constant of proportionality should be universal for islands having dimensions much larger than the superconducting coherence length ξ_0^{Nb} (since no scales aside from E_{Th} are relevant). However, we find that the ratio of T_c to E_{Th} is not universal but depends on the island height, as can be seen in figure 3(d). The dependence T_c of on island height is not monotonic and is likely a signature of the mesoscopic, granular character of the islands [14], as $\xi_0^{Nb} \approx 27$ nm is comparable to the grain size within the islands (though smaller than the island size). Because the island height changes the prefactor but not the $1/d^2$ scaling of the transition temperature, we conjecture on dimensional grounds that

$$\frac{T_c}{E_{Th}} = f\left(\frac{G\delta}{\Delta}\right)$$

where δ is the spacing of energy levels in the grains that constitute each superconducting island, and G is the dimensionless conductance of the normal metal; the product $G\delta$ is thus a measure of the inverse 'dwell time' of an electron on one of the superconducting grains (see, e.g., [22–24]). For the grains discussed here, $\delta \sim 0.8$ μeV, and the conductance of the gold film, $G \sim 950$; thus, the ratio $G\delta/\Delta$ is of order unity, consistent with the appreciable height dependence of T_c that is observed experimentally. In principle, a second candidate for a relevant mesoscopic energy scale is the charging energy on each island. However, the transition temperature of an array of closely spaced islands (e.g., for $d = 90$ nm and $\xi_N(T \approx 8.9$ K$) \approx d$) is approximately equal to the critical temperature an un-patterned, Nb–Au bilayer film, $T_{c,Nb} \approx 8.9$ K. This leads us to believe that the islands are well coupled to the Au film, making it unlikely that charging effects are significant.

To summarize, we have presented systematic measurements of the superconducting transition temperature of SNS arrays as a function of island spacing for widely spaced islands, as well as temperature-dependent critical current measurements to directly establish the key role played by the Thouless energy in determining inter-island phase coherence. The experiments confirm the quasiclassical expectation that the Thouless energy is the scale determining the inter-island coupling; however, our results deviate from quasiclassical expectations in some respects, notably in the dependence of the transition on island height.

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