Fate of the Josephson effect in thin-film superconductors

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Published online: 30 October 2005; doi:10.1038/nphys154

The d.c. Josephson effect refers to the dissipationless electrical current—the supercurrent—that can be sustained across a weak link connecting two bulk superconductors. This effect probes the nature of the superconducting state, which depends crucially on spatial dimensionality. For bulk (that is, three-dimensional) superconductors, the superconductivity is most robust and the Josephson effect is sustained even at non-zero temperature. However, in wires and thin films, thermal and quantum fluctuations play a crucial role. In superconducting wires, these effects qualitatively modify the electrical transport across a weak link. Despite several experiments involving weak links between thin-film superconductors, little theoretical attention has been paid to the electrical conduction in such systems. Here, we analyse the case of two superconducting thin films connected by a point contact. Remarkably, the Josephson effect is absent at non-zero temperature. The point-contact resistance is non-zero and varies with temperature in a nearly activated fashion, with a universal energy barrier set by the superfluid stiffness characterizing the films. This behaviour reflects the subtle nature of thin-film superconductors and should be observable in future experiments.

Soon after the development of the microscopic theory of superconductivity[1], Josephson predicted a remarkable manifestation of it: even in the absence of a voltage difference, a supercurrent of electric charge can flow across a weak link between two bulk superconductors (for example, an insulating barrier)[2]. This remarkable prediction by Josephson was observed at the time by Giaever[3], but was misinterpreted as a 'metallic short'[4]. Both were awarded the Nobel prize in 1973 for their discoveries.

Since its discovery, the Josephson effect has had a major impact on a broad spectrum of technologies. The most sensitive magnetic-flux and electromagnetic-radiation detectors are superconducting quantum interference devices (SQUIDs), consisting of two Josephson junctions connected in parallel. SQUIDs play a crucial role in many condensed-matter experiments and in radioastronomy, and in biomagnetic detectors to monitor brain activity. Josephson junctions were also a stepping stone in computer technology. In the 1970s, the burgeoning semiconductor revolution led IBM to abandon its effort to construct computers out of arrays of Josephson junctions. Today, however, circuits made of Josephson junctions are back in vogue, in the effort to construct a quantum computer[5,6].

Josephson's prediction was based on the fundamental principle of broken symmetry. A superconductor can be thought of in terms of a complex-valued wavefunction (or 'order parameter') $\psi(r)$ for Cooper pairs. At the onset of superconductivity, Cooper pairs undergo Bose condensation and the order parameter becomes non-zero with a well-defined phase φ: $\psi(r)=|\psi|\exp(i\varphi)$. The symmetry of phase rotations is thus broken. When the phase $\varphi(r)$ varies in space, a bulk supercurrent proportional to its gradient results. Similarly, a Josephson supercurrent flows between two superconductors with different phases. It is proportional to the sine of the phase difference across the weak link:

$$I=\frac{2eJ}{\hbar}\sin(\varphi_1-\varphi_2). \qquad (1)$$

Here, J is the Josephson coupling energy, which characterizes the coupling strength between the superconductors, e is the electronic charge and $\hbar$ is the reduced Planck constant.

The Josephson effect was initially measured between two bulk (that is, three-dimensional) superconducting electrodes. Experimental techniques have since evolved significantly; present-day experiments probe superconductivity in nanometre-scale samples, as well as in systems with reduced dimensionality. Particularly intriguing experiments have measured the electrical resistance across nanowire junctions between thin-film superconductors[7,8], through a narrow constriction (about 20 nm wide) between two films as thin as 2.5 nm (ref. 9), and between a film and a superconducting scanning tunnelling tip[10]. In such nanoscale systems with reduced dimensionality the simple Josephson effect of equation (1) must be revisited.

In large bulk superconductors, the phase of the order parameter φ is a rigid, essentially classical, variable. In equilibrium, and in the absence of currents, φ is locked to a single fixed value; the system shows long-range order (LRO). This is true even in the presence of thermal fluctuations, provided the sample is below the superconducting-transition temperature. The phase rigidity of three-dimensional superconductors prevents significant phase fluctuations that would suppress the Josephson supercurrent, and equation (1) holds. It is important to emphasize that these statements hold in the limit of large superconducting electrodes; otherwise, there is neither true LRO nor a true Josephson effect.

In thin-film superconductors, thermal fluctuations of the phase markedly alter this simple picture. Specifically, phase correlations are no longer infinitely long-ranged; instead, the correlator $\langle e^{i\varphi(r_1)}e^{-i\varphi(r_2)}\rangle$ (where $\langle\ldots\rangle$ denotes a thermal average) decays as a power law with spatial separation. This phenomenon is known as quasi-long-range order (QLRO) and occurs below the Berezinskii–Kosterlitz–Thouless phase-transition temperature $T_{\rm BKT}$. Thermal phase fluctuations varying smoothly in space, present because of thermal excitation of the superconducting plasmon mode, are responsible for the power-law phase correlations. Only at $T=0$, where these are frozen out, is LRO present. Remarkably, the resistance of a two-dimensional film vanishes for $T<T_{\rm BKT}$, despite the absence of LRO. Above $T_{\rm BKT}$, however, the superconducting state is disrupted by topological vortex defects, around which $\varphi(r)$ winds by an integer multiple of 2π. A non-zero density of mobile vortices scrambles the phase; only short-ranged superconducting correlations survive and the above correlator decays exponentially with spatial separation. In addition, the resistance of the film is non-zero in this regime. Below $T_{\rm BKT}$, isolated vortices are expelled from the film or tightly bound into vortex–antivortex pairs, and QLRO results. In the following, we consider the fate of the Josephson effect across a point contact separating two films below $T_{\rm BKT}$.

The question of the Josephson effect in systems with reduced dimensionality has been addressed previously, but in the context of one-dimensional superconducting wires. At any non-zero temperature, the phase correlations in one dimension are short-ranged and one expects a non-vanishing resistance even in the absence of a weak link[11,12]. This can be understood in terms of thermally excited phase-slip events. At zero temperature, a one-dimensional superconductor shows QLRO and the resistivity vanishes. This holds provided $g>g_{\rm c}$, where g is the superfluid stiffness measured in appropriate units and is proportional to the cross-section of the wire. The limit $g_{\rm c}$ is of the order of unity and is determined by the nature of any Umklapp scattering that may be present. The effects of a weak link on a one-dimensional superconductor have been addressed theoretically; provided $g>1$, it was found that at zero temperature the weak link 'heals' itself and the resistance across the point contact vanishes (except in the case of a wire with only a single transverse electron mode at the Fermi energy). At small non-zero temperatures, the resistance across the point contact is predicted to vanish as a power law in temperature[13]:

$$R\propto T^{2(g-1)}.$$

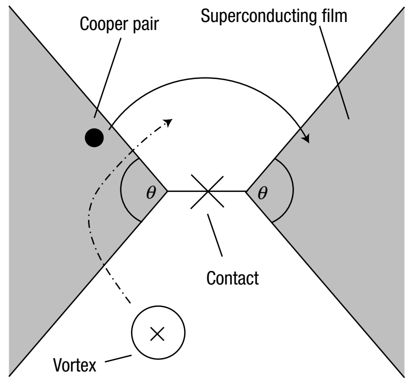

Figure 1 Top view of Cooper pair and vortex tunnelling. Charge transport across the contact can be viewed most directly in terms of Cooper-pair hopping events (solid arrow) with amplitude J. Alternatively, one can consider phase-slip events, where the phase difference between the two films winds by 2π. These events occur when a vortex moves through one of the films near the junction, transverse to the current flow (dash–dot line); this produces a momentary voltage spike between the films. The net amplitude for all such processes, which only occur near the contact, is $t_{\rm v}$.

The Josephson effect is obliterated at non-zero temperature, in marked contrast to the weak link between three-dimensional superconductors. (It should be noted that, although ref. 13 explicitly dealt with a single-channel quantum wire, the effective field theory is identical to that for a many-channel superconducting wire, and the preceding statements follow immediately from the analysis there.)

What is the fate of the Josephson effect in the intermediate case, when the point contact is between two films rather than wires or bulk electrodes? Surprisingly, despite several experiments on two-dimensional films, very little theoretical attention has been paid to this problem (but see ref. 14). In this article we determine the tunnelling resistance across a point contact between two films in the geometry shown in Fig. 1. As it turns out, a weak link between two films is almost superconducting, but a true Josephson effect is absent except at zero temperature. We find that the resistance $R(T)$ drops very rapidly on cooling, in a nearly activated fashion. This behaviour is compared with the known results for three-dimensional and one-dimensional superconducting electrodes in Table 1. More specifically, at low temperatures

$$\frac{R(T)}{R_{\rm Q}}=\frac{t_{\rm v}^2}{\sqrt{E_{\rm A}(T)}}\frac{1}{(k_{\rm B}T)^{3/2}}\exp\left[-\frac{E_{\rm A}(T)}{k_{\rm B}T}\right],\tag{2}$$

where $t_{\rm v}$ is an amplitude for quantum phase-slip processes, discussed in more detail below. This formula is expected to be asymptotically exact at very low temperatures. Here, $R_{\rm Q}=h/4e^2$ is the quantum of resistance, where h is the Planck constant, $k_{\rm B}$ is the Boltzmann constant and $E_{\rm A}(T)$ is a temperature-dependent activation energy:

$$E_{\rm A}(T)=cK_{\rm s}\frac{1}{\ln(\hbar\omega_{\rm c}/2k_{\rm B}T)+(2cK_{\rm s}/\pi^2J)}.$$

$R(T)$ is positive for all non-zero temperatures, but vanishes in an activated fashion as $T\to 0$, up to the logarithmic correction contained in the denominator of $E_{\rm A}(T)$. Thus, in the low-temperature limit $R(T)$ vanishes faster than any power law. The

Table 1 Comparison of the electrode resistivity and tunnelling resistance across a point contact for superconductors of varying spatial dimensionality. The entries of the table describe the behaviour at low temperature. The main result of our work, which completes this table, is the nearly-activated behaviour of the resistance across a point contact between two-dimensional superconductors.

Spatial dimensionality	Resistivity of individual electrodes	Point-contact tunnelling resistance
$d=3$	Zero	Zero
$d=2$	Zero	Nearly activated
$d=1$	Power law	Power law

scale of the activation energy is set by the superfluid stiffness in the two-dimensional films, $K_s=\hbar^2 n_s/m$, where n_s is the density of Cooper pairs in the films and m is the pair mass. The parameter ω_c is a high-energy cutoff frequency discussed further below. The dimensionless number c is given by

$$c=\frac{\pi^2\theta}{4\alpha},$$

where θ is the 'opening angle' shown in Fig. 1 and α is a parameter characterizing the range of the interactions. Specifically, $\alpha=2$ for Coulomb interactions and $\alpha=1$ for screened interactions, as can be obtained in the presence of a superconducting ground plane (see below). Remarkably, the main effect of the Josephson coupling is to set a crossover temperature $T_J\sim(\hbar\omega_c/2k_B)\exp(-2cK_s/\pi^2 J)$. At low temperatures, where the $\ln(1/T)$ term dominates the denominator of $E_A(T)$, that is $T\ll T_J$, the dependence on J disappears altogether, and one recovers the universal result:

$$E_A\sim cK_s/\ln(\hbar\omega_c/2k_BT). \tag{3}$$

The important temperature scale in this limit is set by K_s, a property of the two-dimensional films that can be measured independently[15] and is insensitive to the details of the contact. At higher temperatures, $R(T)$ is approximately given by a purely activated form (that is, having no logarithmic corrections), with the barrier height set by the Josephson coupling: $E_A\approx(\pi^2/2)J$.

The resistance formula of equation (2) is exact at low temperature, in the sense that $\ln[R(T)]/\ln(R_{\text{measured}})\to 1$ as $T\to 0$. The ratio of the resistances themselves does not go to unity, as there are additive corrections to $E_A(T)$ proportional to $\ln^{-2}(1/T)$. These corrections are contained in the integrals leading to equation (2) (see Supplementary Information), so one can improve accuracy by evaluating them exactly, which must be done numerically.

Our result for $R(T)$ fulfills some basic physical requirements. The zero-temperature resistance vanishes, as it should in the presence of LRO in the two-dimensional films. Furthermore, the resistance vanishes faster than any power law as T approaches zero, as is reasonable on comparison with the one-dimensional and three-dimensional cases. Finally, the activation energy increases monotonically with increasing Josephson coupling.

Before addressing the experimental implications of our result, we briefly discuss its derivation and the underlying physics. The thin-film superconducting electrodes and the point contact are modelled by a quantum-phase hamiltonian, which focuses on the quantum and thermal fluctuations of $\varphi(r)$. This is legitimate at temperatures well below the quasiparticle gap scale and T_{BKT}. The hamiltonian for a single thin-film electrode is

$$H_{\text{film}}=\frac{K_s}{2}\int d^2r(\nabla\varphi)^2+\frac{1}{2}\int d^2r d^2r' n(r)n(r')V(r-r'),$$

where $n(r)$ is the fluctuating Cooper-pair density, canonically conjugate to $\varphi(r)$. The spatial integrations range over the area of the film. The first term encodes the energy cost for phase gradients; this is essentially the kinetic energy of the superflow. The second term is a density–density interaction with potential $V(r)$; we consider both Coulomb ($V(r)=(2e)^2/|r|$) and screened ($V(r)=(\hbar^2v_s^2/K_s)\delta(r-r')$) interactions, where v_s is the plasmon velocity and $\delta(r)$ is the Dirac delta function. This hamiltonian describes the quantum dynamics of the plasmon mode of the superconductor, which is characterized by its frequency ω as a function of the wavevector k. In the case of screened interactions, there is a linearly dispersing acoustic plasmon ($\omega=v_sk$), whereas for Coulomb interactions the plasmon has the dispersion $\hbar\omega=\sqrt{8\pi e^2K_sk}$. The high-energy cutoff ω_c appearing in equation (2) is given in terms of this dispersion: $\omega_c=\omega(k=2\pi/\xi)$, where ξ is a short-distance cutoff of the order of the superconducting coherence length.

The second element in our system is the weak link, which we shall refer to as a point contact. We model it as a point-like Josephson coupling between the two electrodes and set the coordinate origin for both films (that is, $r=0$) at the contact:

$$H_{\text{contact}}=-J\cos(\varphi_2(0)-\varphi_1(0)).$$

Here, $\varphi_{1,2}(r)$ are the phase fields in the two electrodes. This term can be interpreted as a process hopping Cooper pairs across the contact, where J determines the hopping amplitude. It should be noted that we have neglected inter-film Coulomb interactions. Because any geometric zero-frequency capacitance will diverge with the size of the films, Coulomb blockade effects will be unimportant, except below an extremely low temperature. Above this temperature, we do not expect inter-film interactions to modify the low-temperature resistance.

In the limit $J\ll K_s$ (that is, a poor contact), we may calculate the current response to a small voltage bias in terms of the hopping of Cooper pairs across the contact. Specifically, one can expand in powers of J to calculate the conductance G across the point contact: $G(T)=a_2J^2+O(J^4)$. It has been found[14] that the coefficient $a_2(T)$ is divergent for temperatures below $T^*=\theta K_s/2\alpha k_B$, and the conclusion drawn that the conductance diverges (and hence the resistance vanishes) for $T<T^*$. However, the physical meaning of this result is unclear; in fact, we shall show that it indicates not zero resistance but rather a breakdown of perturbation theory. More information, and an independent test of this perturbative result, can be obtained through an expansion in the opposite limit: that of a good contact. Indeed, the above result already suggests a strong tendency towards this limit and justifies the approach taken below.

The limit of a good contact is readily accessed through a dual picture. Here, we begin by assuming perfect phase-coherence across the contact. This coherence is then weakened by quantum phase-slip events, which occur when a vortex tunnels across the film near the contact, in the direction transverse to the current flow (as illustrated in Fig. 1). A phase slip makes the relative phase of the electrodes wind by 2π; this 2π twist can either heal locally by a phase slip in the opposite direction or propagate outward into the electrodes. The latter process will register as a voltage spike across the system, owing to the Josephson relation

$$\Delta V=\frac{1}{2e}\frac{d}{dt}\Delta\varphi,$$

where ΔV and $\Delta\varphi$ are the voltage and phase differences, respectively.

In this dual picture the resistance can be obtained through a perturbation expansion in t_v, which is the amplitude for a phase slip to occur or, equivalently, for a vortex to hop through the contact. Formally, we integrate out the degrees of freedom in the films to obtain an action for the phase difference $\phi = \varphi_1(0) - \varphi_2(0)$ across the contact. The dual action for the phase slips can then be obtained by a Villain transformation[16]. The phase slips are instanton events where ϕ jumps by $\pm 2\pi$, and we define t_v as their fugacity. The single vortex-hopping process with amplitude t_v should be viewed as an encapsulation of the many different physical vortex-hopping processes in the vicinity of the contact (see Fig. 1). We note that t_v depends only weakly on temperature (it goes to a constant at $T = 0$). It also has an implicit dependence on J, decreasing as J increases. It would be interesting to compute this dependence, but we do not attempt to do so here.

Following the above approach, we have calculated the voltage response to a vanishingly small current across the point contact, obtaining the resistance formula equation (2) at order t_v^2. This formula encapsulates the exact low-temperature dependence of the t_v^2 term and is quantitatively accurate over a reasonably large temperature range. As an example, using the parameters for the system of MoGe films with ground plane discussed below, one can compare $R(T)$ with the exact resistance at order t_v^2 (obtained numerically). The logarithms of these quantities agree within 15% for $T \le 4$ K. Furthermore, contributions of higher order in t_v involve larger effective activation barriers and are therefore unimportant at low temperatures. This is to be expected, as these terms correspond to processes involving more phase-slip events and the phase slips are strongly suppressed by the QLRO in the films.

The resistance formula obtained from the dual calculation is expected to be exact at low temperatures, even in the case of a poor contact. Physically, the expansion in phase-slip events is valid in the limit of small resistance. As $R(T = 0)$ vanishes because of the presence of LRO in the films, the phase-slip expansion is always correct at low temperatures. Reasoning along the same lines, our result should be a good approximation as long as the argument of the exponential in equation (2) is large and negative.

The most interesting regime is the universal low-temperature limit, that is equation (3). This regime will be more easily accessible when J/K_s is rather large; this could be achieved by fabricating the point contact as a short and wide constriction.

The most important quantity determining the universal low-temperature resistance is the superfluid stiffness K_s. This can be obtained directly for thin films by an inductance measurement[15]. In fact, one could make a direct check of our theory by varying K_s *in situ* with an in-plane magnetic field. The other relevant parameters are the Josephson coupling J and the cutoff frequency ω_c, which should be considered as fitting parameters. In many cases it should be possible to estimate J from K_s and the geometry of the contact (see below). An estimate of ω_c is given in terms of the coherence length in the films, as discussed above.

Our result should be accessible to experimental tests in a variety of different systems. Here we consider the specific case, which has been fabricated and studied by Chu *et al.*[9], of a narrow constriction between two MoGe films. We imagine a modification of their setup, in which a superconducting ground plane with stiffness K_s^g is added a distance d below the two-dimensional films, which we take to be 100 nm. We consider $K_s^g \gg K_s$ as appropriate for a thick ground plane, to screen out the Coulomb interactions as much as possible. This setup leads to an acoustic plasmon propagating in the films with velocity $v_s = (1/\hbar)\sqrt{16\pi e^2 K_s d}$. There is also a plasmon with $\omega \propto \sqrt{k}$ dispersion, which propagates within the ground plane and does not contribute to the tunnelling transport. The constrictions in the experiments of ref. 9 had width $w \approx 20$ nm and length $\ell \approx 100$ nm. The films themselves show a superconducting transition at $T_{BKT} \approx 5$ K and have a coherence length $\xi \approx 7$ nm. We consider $\theta = \pi$ and would have $\alpha = 1$ with the screening ground plane in place. For illustrative purposes, we assume $K_s/k_B \approx 10$ K for $T \ll T_{BKT}$, and then $\hbar\omega_c/k_B \approx 2\pi\hbar v_s/k_B d \approx 2{,}000$ K. (In the presence of the ground plane, the short-distance cutoff in the films is set by d rather than ξ.) We estimate $J/k_B \approx K_s w/\ell k_B \approx 2$ K. For these parameters, neglecting the temperature dependence of K_s, the argument of the exponential in $R(T)$ is -1 for $T \approx 5$ K, so we conclude that our formula should be good for $T \lesssim 5$ K. We note that the resistance may be fairly small at the lowest accessible temperatures; in this regime it may perhaps be measured most accurately through the decay of the persistent current in a ring geometry.

One of the most important considerations for experiments is that the superconducting films be in the thermodynamic limit. For a film of linear dimension L, we can define a crossover temperature scale $k_B T_L = \hbar\omega(k = 2\pi/L)$, which vanishes as the linear size L of the films diverges. When $T \gg T_L$, thermal processes prevent quanta of the plasmon mode from travelling coherently between the film edges. This thermal decoherence washes out the influence of the environment (and other finite-size effects) on the system of films and point contact. With a superconducting ground plane as above, one has $k_B T_L = 2\pi\hbar v_s/L$. For $L = 1$ cm we find $T_L \approx 2 \times 10^{-2}$ K. This leaves a broad temperature range over which the resistance formula equation (2) should apply and also points out that it is important to have rather large films to lower T_L. It is difficult to achieve such a broad temperature range without the ground plane, as T_L is only proportional to $L^{-1/2}$ in that case; for the parameters as above, but with no ground plane, $T_L \approx 1$ K. However, surrounding the film by a medium with rather large dielectric constant ϵ would lead to a modest improvement, because $T_L \propto \epsilon^{-1/2}$.

It is interesting to point out that the system studied here cannot be understood in terms of the phenomenological resistively and capacitively shunted junction (RCSJ) model, which is one of the standard theoretical approaches to Josephson junction physics[17,18]. In that model, one imagines that the junction is shunted by a parallel resistor and capacitor. One then writes down a hamiltonian in terms of only ϕ, the phase difference across the junction, and N, a canonically conjugate integer-valued Cooper-pair density. This takes the form

$$H_{RCSJ} = \frac{1}{2C}N^2 - J\cos(\phi),$$

where C is the capacitance. The resistor is a phenomenological model for the dissipation in the system, which is assumed to be of the Caldeira–Leggett form[19] appropriate for dissipation owing to the plasmons of a one-dimensional superconductor[13], or the similar Ambegaokar–Eckern–Schön form[20], which models dissipation owing to a Fermi surface of gapless quasiparticles or electrons. The RCSJ model does not apply, however, for point contacts between large two-dimensional or three-dimensional superconducting leads with gapped quasiparticles. In these cases, the capacitance diverges as the size of the leads is taken to infinity. The charging energy vanishes in this limit, and Coulomb-blockade effects disappear above a temperature scale proportional to $1/L$ for both two-dimensional and three-dimensional cases with Coulomb interactions. As for the resistor, in these systems there is either dissipation owing to the superconducting plasmons in the two-dimensional case, or a pinning effect owing to LRO in the three-dimensional case (see Supplementary Information). These effects cannot be modelled by dissipation of the Caldeira–Leggett or Ambegaokar–Eckern–Schön form, as they are much stronger at low frequency. Roughly speaking, the effect of the two-dimensional or three-dimensional electrodes is much closer to connecting the junction to a pair of ground planes, rather than to a resistor. Indeed, even if one adds a physical shunt resistor to the system, at low frequencies it will be shorted out by the ground planes.

Finally, we expect that our result will describe transport in a variety of systems—our model only requires that the low-energy excitations in the films can be described by some bosonic degrees of freedom governed by an XY model in its QLRO phase. This physics is most readily accessible in two-dimensional superconducting films, and we hope our work will stimulate more experiments on these intriguing systems.

Received 17 August 2005; accepted 3 October 2005; published 30 October 2005.

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Acknowledgements

We gratefully acknowledge discussions with L. Balents, A. Bezryadin, A. Paramekanti and X.-G. Wen. This research is funded by the US Department of Defense NDSEG program (M.H.), the US National Science Foundation (G.R. and M.P.A.F.) and the US Department of Energy, Division of Material Sciences (through the Frederick Seitz Materials Research Laboratory at UIUC) (P.G.).
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Supplementary Information accompanies this paper on www.nature.com/naturephysics.

Competing financial interests

The authors declare that they have no competing financial interests.