

QUANTUM COHERENCE

Just what is superconductivity?

Whether or not a superconductor is truly superconducting depends on its size and even its shape. In a geometry intermediate between one and two dimensions, it seems a thin film does not reach a state of zero resistance except at zero temperature.

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Superconductivity seems like a magic example of quantum mechanics on a macroscopic scale. At low temperatures, certain materials undergo a thermodynamic phase transition at which they apparently lose all resistance to the flow of electricity and tend to expel any magnetic fields inside them. This vanishing of resistance permits currents in wire loops to persist almost indefinitely, and is the basis of important magnet technologies. On page 117 of this issue¹, Michael Hermele *et al.* discuss a novel 'bowtie' geometry for superconducting thin films in which they predict that the electrical resistance will be almost — but not quite — zero even at very low temperatures.

Many systems undergo phase transitions in which they change from a disordered state to an ordered state as the temperature is lowered. The vacuum of space itself is a turbulent quark–gluon plasma with unconfined quarks at incredibly high temperatures, but at ordinary temperatures, the ordered state of the vacuum excludes colour electric fields, leading to quark confinement. This effect is analogous to the exclusion of magnetic fields by superconductors and both are examples of a kind of 'rigidity' closely analogous to the literal rigidity that develops in water when it undergoes the freezing transition. For example, a person standing on a frozen lake will never sink into the ice as long as the molecules of water remain in the ordered state of the solid. The order characterizing a solid is easy to understand. Once we know the position of a few atoms, we can predict the positions of all the others, because they form a regular lattice. This ordering is what accounts for the rigidity, defined as the ability to transmit shear forces from one end of the solid to the other — something a liquid cannot do.

The ordering in a superconductor is more subtle. Pairs of electrons with opposite spin condense into a single macroscopic wavefunction whose complex phase can be represented pictorially by an array of arrows inside the sample

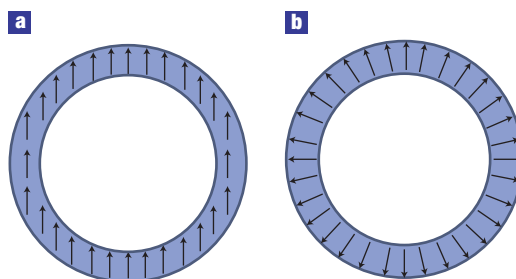


Figure 1 Topological stability.

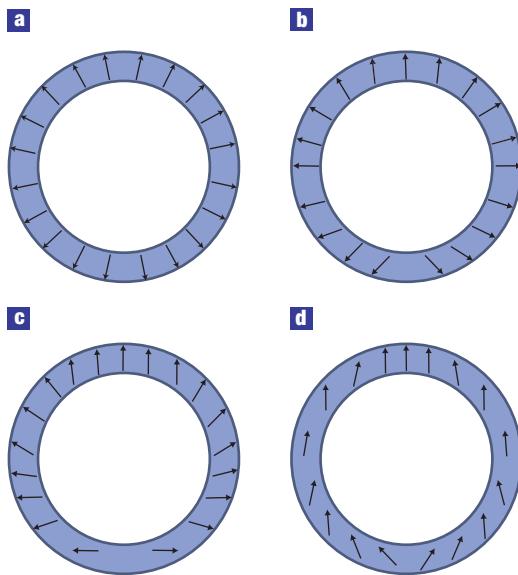
a, A two-dimensional superconducting ring with all the arrows in phase. Turning one of the arrows would result in all the others following its lead, so that they remain in phase. **b**, The phase of the arrows twists by 360° in a clockwise (or anti-clockwise) direction. There is no smooth transition from state **a** to state **b**.

as shown in Fig. 1a. There is an energy that tries to align each arrow with its neighbours. At low-enough temperatures this energy wins out over entropy and the arrows align. The actual orientation of the arrows does not affect the energy, as long as they are all nearly the same. Once we know the orientation of the arrows in one part of the sample, we can predict the orientation in distant parts of the sample. This ordering gives rise to a kind of stiffness in which twisting the arrows at one end of the sample causes the arrows everywhere to turn and follow.

To see how this stiffness leads to persistent currents, consider the configuration of arrows shown in Fig. 1b. Here the arrows are not quite locally parallel because their orientation winds around a full circle as one moves around the ring. This twist of the phase arrows results in extra energy, which, it turns out, represents the fact that current is circulating in the ring (the direction of the current being determined by which way the arrows wind). For this current to decay the arrows must return to the low-energy configuration shown in Fig. 1a. There is no smooth continuous twisting of the arrows that can change the topology of Fig. 1b to that of Fig. 1a. As can be seen in Fig. 2, there is necessarily a singularity at some point where the arrows twist very rapidly (that is, neighbouring arrows are antiparallel) as a topological defect known as a vortex passes across the ring at that point. This imposes an energy barrier that makes it very difficult for the current to decay.

The key question considered by Hermele *et al.* is just how difficult is it for the current to decay? If the wire is three dimensional, the

Figure 2 Unwinding the phase twist. **a**, Going from a state with twisted phases through intermediate states **b** and **c** to reach the phase-aligned state of **d** requires passing through a discontinuity. This singularity, which corresponds to a vortex passing through the ring, comes with an energy cost that prevents current decay.



energy barrier will become arbitrarily large as the diameter of the wire increases, the rate at which thermal fluctuations can overcome this barrier will become arbitrarily slow, and the current will persist essentially forever. If the wire is effectively one-dimensional (very long but of small fixed cross-section) quantum tunnelling of the vortices makes the energy barrier effectively smaller, leading to a finite resistance that will vanish only slowly (as a power law) as the temperature is lowered to zero. Two-dimensional films are an intermediate case. The film resistance goes to zero below some critical temperature but (because the phase arrows are never perfectly ordered) the amount of supercurrent that a film can support is, strictly speaking, infinitesimal.

Hermele and co-workers consider a novel geometry (see Fig. 3) in which two large superconducting film wedges with opening angle θ are joined at the corners. At this thin spot, it would seem to be relatively easy for vortices to pass through the sample and cause finite resistance. However, detailed calculations show that the physics is in a

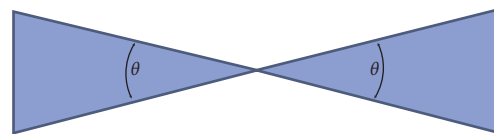


Figure 3 The bowtie geometry used in the calculation. Two two-dimensional thin-film superconductors are connected by a point, with the angle θ controlling the shape of the bowtie.

sense intermediate between that of a one- and a two-dimensional system. The resistance is finite at any finite temperature. But, as the film width grows with distance from the corner, the resistance vanishes much more rapidly than in the one-dimensional case, proportional to an exponential $\exp(-T_0/T)$, where the characteristic temperature scale T_0 is a measure of the energy barrier, and is equal to the stiffness energy that tries to align the phase arrows in the film, multiplied by a universal factor, which depends only on the opening angle θ .

For experimental tests of this prediction, it will be essential to choose the film thickness and the disorder carefully, so that the temperature scale T_0 is large enough that it can be conveniently reached with standard cryogenic techniques, yet not so large that the resistance is immeasurably small. Also, as the characteristic quantum and thermal fluctuations of the phase arrows will occur in the microwave region of frequency, careful engineering of the high-frequency impedance (and thermal noise) presented to the junction by the electromagnetic circuit environment will be essential. Indeed, the central theoretical result of the paper can be re-expressed in terms of weak tunnelling of vortices through the narrow part of the sample in the presence of a frequency-dependent environmental impedance determined by the wedge opening angle.

REFERENCES

1. Hermele, M., Refael, G., Fisher, M. P. A. & Goldbart, P. M. *Nature Phys.* **1**, 117–121 (2005).